

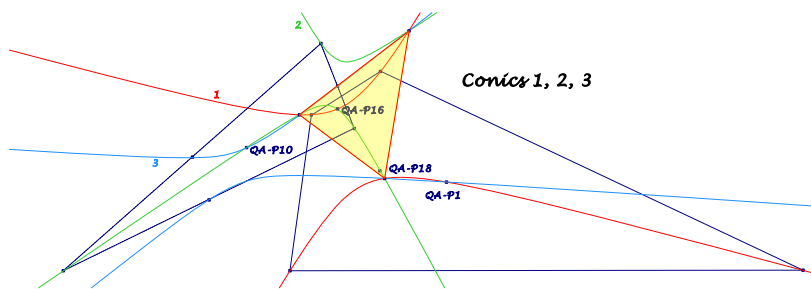
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Two Corresponding Triangles for Quadrangle and Quadrilateral

QA-Triangle

Reference triangle for a quadrangle is the QA-Diagonal Triangle *QA-Tr1* with a QA-isoconjugation (*QA-Tf2*) and fixpoints in the vertices of the quadrangle.

The vertices of the treated QA-triangle are the common points of the following conics:

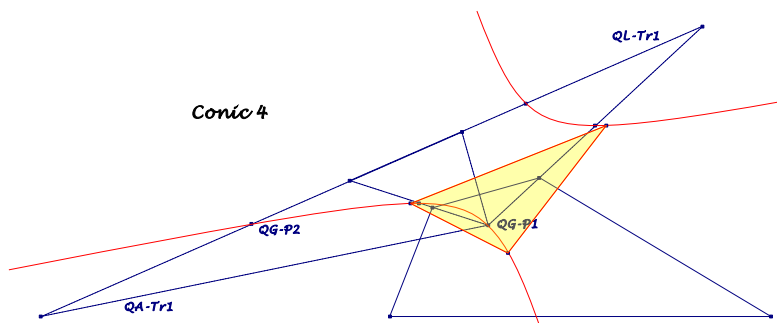


1. Circumscribed conic of the quadrangle through the points *QA-P1* and *QA-P16* (see *EQF-Note 2013-01-18*):

$$q^2 r^2 (q^2 - r^2) x^2 + r^2 p^2 (r^2 - p^2) y^2 + p^2 q^2 (p^2 - q^2) z^2 = 0$$
2. Circumscribed conic of the reference triangle through *QA-P10*, *QA-P16*, *QA-P18*.

$$p^2 (q^2 - r^2) yz + q^2 (r^2 - p^2) zx + r^2 (p^2 - q^2) xy = 0$$
3. Circumscribed conic of the medial reference triangle through *QA-P1* and *QA-P16*.

$$(q^2 - r^2)(x^2 + yz) + (r^2 - p^2)(y^2 + zx) + (p^2 - q^2)(z^2 + xy) = 0$$



4. The following three conics for the quadrigon components: Circumscribed conic of the medial triangle of *QL-Tr1* through *QG-P1* and *QG-P2*.

$$p^2(-x+y+z)y - q^2(x-y+z)x = 0$$

$$q^2(x-y+z)z - r^2(x+y-z)y = 0$$

$$r^2(x+y-z)x - p^2(-x+y+z)z = 0$$

A calculation for the barycentric coordinates of the vertices of this *QA*-triangle needs solutions for equations of degree 3. The following properties are only verified with CABRI.

Properties:

- The trilinear polars of the triangle vertices are parallel.
Point at infinity: $(p^2(q^2-r^2):q^2(r^2-p^2):r^2(p^2-q^2))$
(All points of conic 2 have parallel trilinear polars.)

- The trilinear poles of the triangle sides lie on *QA-P10*.*QA-P16* in the images of the opposite vertices wrt the *QA*-isoconjugation.

$$(q^2-r^2)x + (r^2-p^2)y + (p^2-q^2)z = 0$$

- The isotomic conjugates of the triangle vertices are collinear on a line which is the isotomic conjugate of the conic 2.

$$p^2(q^2-r^2)x + q^2(r^2-p^2)y + r^2(p^2-q^2)z = 0$$

(This line is the trilinear polar of the isotomic conjugate of the point at infinity above.)

- The images of the triangle vertices wrt the *QA*-isoconjugation, which are the trilinear poles of the opposite sides (see above), are the isotomic conjugates of their anti-complements.

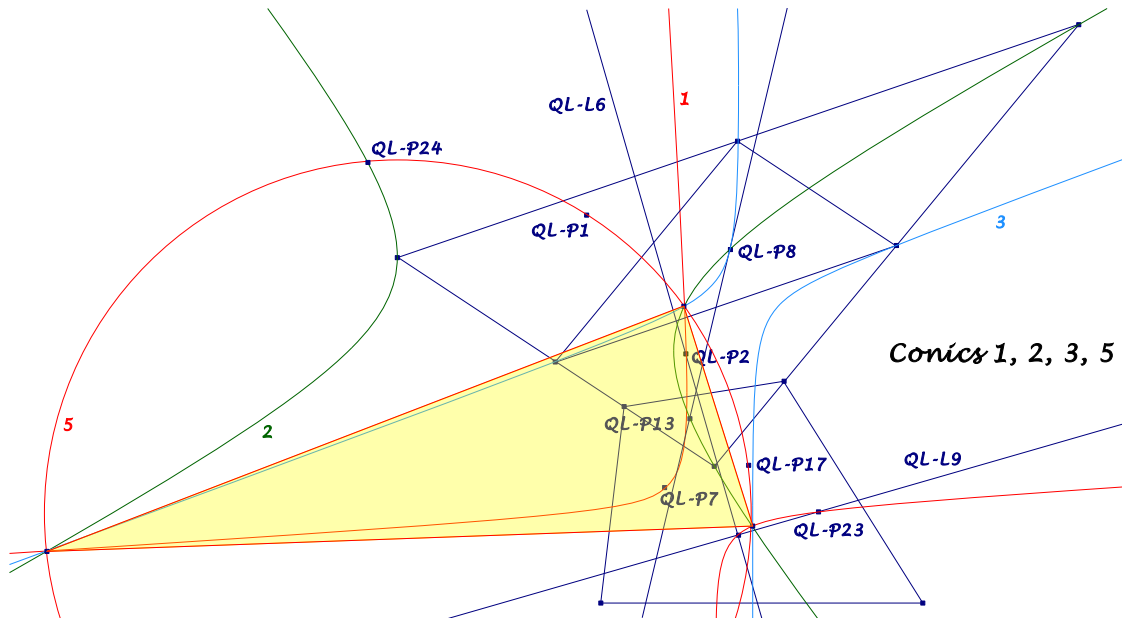
***QL*-Triangle**

Reference Triangle for a quadrilateral is the *QL*-Diagonal Triangle *QL-Tr1*. There is no isoconjugation wrt *QL-Tr1* in *EQF* up to now. But we can define an *QL*-isoconjugation wrt *QL-Tr1* with fixpoints in the trilinear poles of the sidelines of the quadrilateral:

$$(x:y:z) \rightarrow (m^2n^2yz:n^2l^2zx:l^2m^2xy).$$

The here treated *QL*-triangle is already mentioned in *EQF* as $S_1S_2S_3$ in *QL-Ci6*.

The vertices of this *QL*-triangle are the common points of the following conics:



1. Orthogonal hyperbola through $QL-P2$, $QL-P7$, $QL-P23$ and the intersection of $QL-L6$ and $QL-L9$.

$$\sum_{cycl} [((a^2 - 2c^2)l^2m^2 - (a^2 - 2b^2)l^2n^2 - (b^2m^2 - c^2n^2)(m^2 + n^2))l^2x^2 - (2m^2n^2(b^2(l^2 - m^2) + c^2(n^2 - l^2)) - (m^2 - n^2)l^2(-a^2l^2 + b^2m^2 + c^2n^2))yz] = 0$$

2. Circumscribed conic of the reference triangle through $QL-P8$, $QL-P13$, $QL-P24$. This conic is the locus of the trilinear poles of parallels to $QL-L1$.

$$(l^2 - m^2)xy + (m^2 - n^2)yz + (n^2 - l^2)zx = 0$$

3. Isotomic conjugate of $QL-P8.QL-P13$ wrt the medial triangle of the reference triangle.

$$(l^2 - m^2)n^2(xy + z^2) + (m^2 - n^2)l^2(yz + x^2) + (n^2 - l^2)m^2(zx + y^2) = 0$$

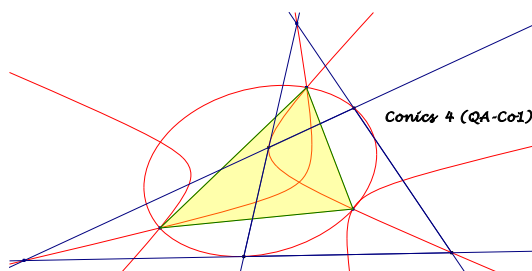
4. The three Nine-Point Conics $QA-Co1$ for the quadrigon components (see $EQF: QL-Ci6$).

$$l^2x^2 + m^2(x - z)y - n^2z^2 = 0$$

$$m^2y^2 + n^2(y - x)z - l^2x^2 = 0$$

$$n^2z^2 + l^2(z - y)x - m^2y^2 = 0$$

5. Dimidium Circle $QL-Ci6$ through $QL-P1$, $QL-P17$, $QL-P24$ (see $EQF: QL-Ci6$).



A calculation for the barycentric coordinates of the vertices of this QL -triangle needs solutions for equations of degree 3. The following properties are only verified with CABRI.

Properties:

- Euler line of the QL -triangle: Centroid $QL-P12$, circumcenter $QL-P6$, orthocenter $QL-P2$.
- The trilinear polars of the triangle vertices are parallel to $QL-L1$. (This property holds for all points of conic 2.)
- The trilinear poles of the triangle sides lie on $QL-P8$. $QL-P13$ in the images of the opposite vertices wrt the QL -isoconjugation.
- The isotomic conjugates of the triangle vertices are collinear on a line, which is the isotomic conjugate of conic 2. (This line is the trilinear polar for the isotomic conjugate of the point at infinity of $QL-L1$.)
- The images of the triangle vertices wrt the QL -isoconjugation, which are the trilinear poles of the opposite sides (see above), are the isotomic conjugates of their anti-complements.

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de