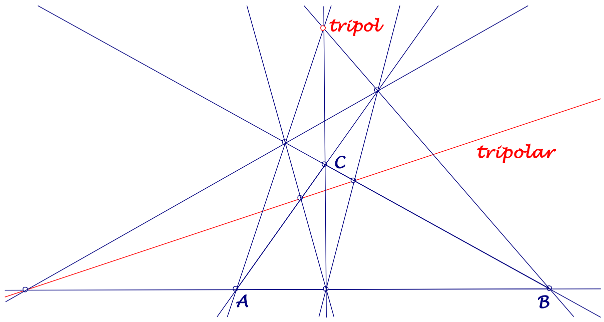
EQF-Note 2013-02-15

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Trilinear Poles and Polars wrt the Diagonal Triangles of Quadrangle, Quadrilateral and Quadrigon

The trilinear polar of *P* (shortened tripolar T_P) wrt a reference triangle *ABC* is the perspectrix of the Cevian triangle of *P* and *ABC*. The trilinear pole of a line *L* (shortened tripol T_L) is the perspector of *ABC* and the triangle of the fourth harmonic points on the side lines of *ABC* wrt the intersection points of *L*.

$$\textit{Tripolar of } P(u:v:w): \quad (vw, wu, uv)$$
$$\textit{Tripol of } L(\lambda,\mu,\nu): \quad (\mu\nu:\nu\lambda:\lambda\mu)$$



- The tripol of the Newton Line *QL-L1* wrt *QL-Tr1* is the *QL*-Harmonic center *QL-P13*.

1. Considering the line pencil of a point *P(u:v:w)*, the locus of the tripols is a circumscribed conic of the reference triangle with the equation

$$wxy + uyz + vzx = 0,$$

centered in $(u(-u+v+w), v(u-v+w), w(u+v-w))$.

The polar of *P* wrt this conic is the tripolar of *P*.

On the other hand: The tripolars of points on a circumscribed conic of the reference triangle have a common point. For example the tripolars of points on the circumcircle concur in the Lemoine point *X(6)*.

1.1 Taking *QL*-points wrt *QL-Tr1*, we get circumscribed conics of *QL-Tr1*, for example:

- *QL-P8* gives the Steiner ellipse with the tripols of all lines through *QL-P8*, e.g. *QL-L7*, *QL-L8*.

- The point at infinity of *QL-L1* gives a circumscribed conic through *QL-P8*, *QL-P13*, *QL-P24* with the tripols of all parallels to *QL-L1*:

$$(l^2-m^2)xy+(m^2-n^2)yz+(n^2-l^2)zx=0\,.$$

1.2 Taking *QA*-points wrt *QA-Tr1*, we get circumscribed conics of *QA-Tr1*, for example:

- *QA-P16* gives the Nine-point Conic *QA-Co1*.
- The tripolars of *QA-P1, QA-P16, QA-P17, QA-P19, QA-P20* (all points of *QA-Co5*) have a common point (not listened in *EQF*):

$$(p^2(q^2-r^2)(-p^2+q^2+r^2), q^2(r^2-p^2)(p^2-q^2+r^2), r^2(p^2-q^2)(p^2+q^2-r^2))\,.$$

2. Considering points on a Line *L*(λ,μ,ν), the tripolars will envelope an inscribed conic of the reference triangle with the equation

$$\lambda^2x^2+\mu^2y^2+\nu^2z^2-2(\lambda\mu xy+\mu\nu yz+\nu\lambda zx)=0$$

centered in $(\mu+\nu,\nu+\lambda,\lambda+\mu)$.

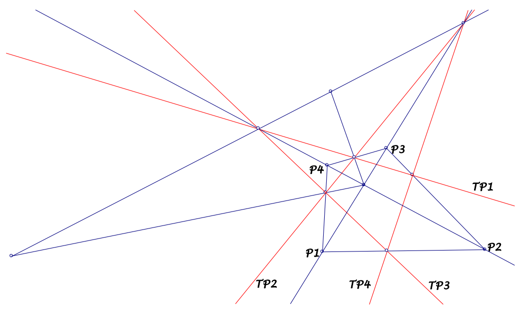
The pole of *L* wrt this conic is the tripol of *L*.

On the other hand: The tripols of tangents at an inscribed conic of the reference triangle are collinear. The tripols of tangents at the incircle lie on the tripolar of the Gergonne Point *X(7)*.

2.1 Taking *QA*-lines wrt *QA-Tr1*, we get *QA-Tr1* inscribed conics (no examples).
2.2 Taking *QL*-lines wrt *QL-Tr1*, we get *QL-Tr1* inscribed conics (no examples).

3. Of special interest are the tripolars of the vertices of a quadrangle wrt *QA-Tr1* and the tripols of the side lines of a quadrilateral wrt *QL-Tr1*.

3.1. The tripolars T_{Pi} of the vertices of a quadrangle give a quadrilateral.

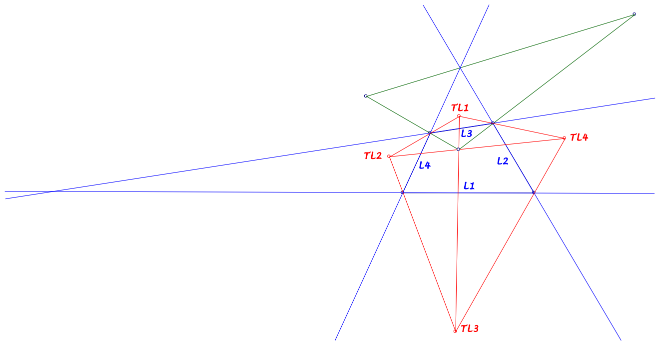


- The *QL*-Diagonal Triangle of this quadrilateral is the *QA*-Diagonal Triangle of the reference quadrangle (evidently with the same Euler configuration).

- Another correspondence: *QL-P13* of this quadrilateral is *QA-P16* of the reference quadrangle.

3.2 The tripols T_{Li} of the side lines of a quadrilateral give a quadrangle.

- The *QA*-Diagonal Triangle of this quadrangle is the *QL*-Diagonal Triangle of the reference quadrilateral (evidently with the same Euler configuration).
- Another correspondence: *QA-P16* of this quadrangle is *QL-P13* of the reference quadrilateral (see above).



4. The tripols of the side lines of a quadrilateral wrt *QL-Tr1* can be taken for fixpoints of an isoconjugation with reference triangle *QL-Tr1* (see EQF-Note 2013-01-01):

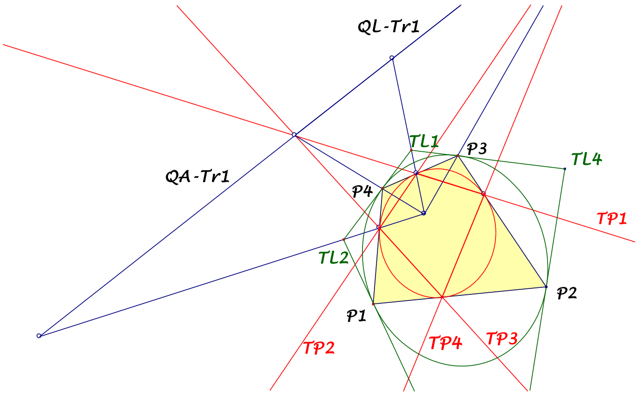
$$(x:y:z) \;\rightarrow\; (m^2n^2yz : n^2l^2zx : l^2m^2xy)\,.$$

- This isoconjugation swaps *QL-P8* and *QL-P13*.
- This isoconjugation transforms lines $L(\lambda,\mu,\nu)$ into *QL-Tr1* circumscribed conics with equation:
$$m^2n^2\lambda\, yz + n^2l^2\mu\, zx + l^2m^2\nu\, xy = 0\,.$$
- Otherwise: Taking the isoconjugate of the tripol of the line $L(\lambda,\mu,\nu)$, the line pencil of this point will give with its tripols the same conic.
- For example: The isoconjugation transforms *QL-L1* in the Steiner circumscribed ellipse; the points of this conic have tripolars through *QL-P8* (see above).
- Another example (see EQF-Note 2013-01-01): This isoconjugation transforms the line *QL-P8.QL-P13* in a *QL-Tr1* circumscribed conic through the points *QL-P8, Ql-P13, QL-P24*; the points of this conic have tripolars parallel to *QL-L1*.

The tripolars of the vertices of a quadrangle wrt *QA-Tr1* can be taken for fixlines of an “isoconjugation for lines” with reference triangle *QA-Tr1*, but no mentionable examples can be found.

5. For a quadrigon there is a first quadrigon $T_{L1}T_{L2}T_{L3}T_{L4}$ of the tripols of the side lines wrt *QL-Tr1* and a second quadrigon $T_{P1}T_{P2}T_{P3}T_{P4}$ of the tripolars of the vertices wrt *QA-Tr1*.

- The T_L- and the T_P-quadrigon have the same *QA*- and *QL*-Diagonal Triangle, that are the *QL*- and the *QA*-Diagonal Triangle of the reference quadrigon.
- The vertices of the *TL*-quadrigon are the intersections of the tangents of *QG-Co2* in the vertices of the reference quadrigon.
- The vertices of the *TP*-quadrigon are the touch points of *QG-Co1* and the side lines of the reference quadrigon.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de