EQF-Note 2013-02-16

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Circumscribed Conics of a Quadrangle

It is obvious to define circumscribed conics of a quadrangle by taking a fifth point in addition to the four vertices of a quadrangle. Nevertheless there are only two circumconics of a quadrangle in EQF up to now: The QA-Orthogonal Hyperbola QA-Co2 (centered in QA-P2) and the Gergonne-Steiner Conic QA-Co3 (centered in QA-P3). Unfortunately these conics contain no registered QA-points. In EQF-note 2013-01-18 there are mentioned two other circumconics: The QA-P5-QA-P10- and the QA-P1-QA-P16-circumconic. − For an arbitrary circumconic here will be developed a related circumconic, using trilinear polars wrt the triangle components of a quadrangle. – Reference triangle for barycentric coordinates will be the QA-Diagonal Triangle QA-Tr1.

Trilinear Poles and Polars

The trilinear polar of *P* wrt a reference triangle *ABC* is the perspectrix of the Cevian triangle of *P* and *ABC*. The trilinear pole of a line *L* is the perspector of *ABC* and the triangle of the fourth harmonic points on the side lines of *ABC* wrt the intersection points of *L*.

$$\textit{Tripolar of } P(u:v:w): \quad (vw, wu, uv)$$
$$\textit{Tripol of } L(\lambda,\mu,\nu): \quad (\mu\nu:\nu\lambda:\lambda\mu) .$$

Considering the line pencil of a point *P(u:v:w)*, the locus of the trilinear poles is a circumscribed conic of the reference triangle with the equation

$$wxy + uyz + vzx = 0 ,$$

centered in $(u(-u+v+w), v(u-v+w), w(u+v-w))$.

The polar of *P* wrt this conic is the trilinear polar of *P*.

On the other hand:

The trilinear polars of points on a circumscribed conic of the reference triangle have a common point.

For example the trilinear polars of points on the circumcircle concur in the Lemoine point *X(6)*. For further properties see *EQF*-note 2013-02-15. In that note the reference triangle is *QA-Tr1*, here we shall use for reference triangles the triangle components of a quadrigon.

Related Circumconics

We start with a quadrangle and a fifth point P(u:v:w) and consider the circumconic through P_1, P_2, P_3, P_4, P with the equation (reference triangle *QA-Tr1*)

$$Px^2+Qy^2+Rz^2=0$$

$$with \quad P=q^2w^2-r^2v^2, \quad Q=r^2u^2-p^2w^2, \quad R=p^2v^2-q^2u^2 .$$

Center of this circumconic is $(QR:RP:PQ)$.

This first circumconic is evidently a circumconic for the triangle components $P_1P_2P_3$, $P_2P_3P_4$, $P_3P_4P_1$, $P_4P_1P_2$. So the trilinear polars of points on this circumconic wrt one triangle component have a common point. For example wrt the residue triangle of $(p:q:r)$ this common point is $(p^3P:q^3Q:r^3R)$. In this way we get a second quadrangle with the property:

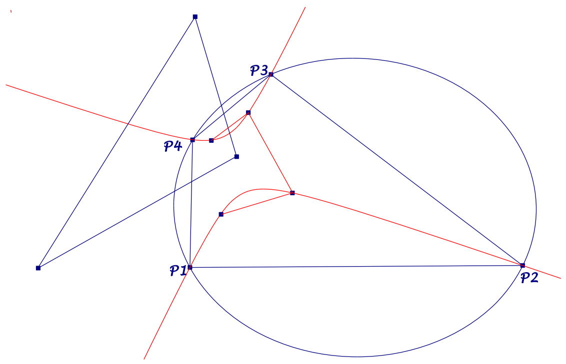
The trilinear polars of points on a *QA*-circumconic wrt the triangle components define a second quadrangle with the same diagonal triangle.

The vertices of the second quadrangle lie on a second *QA*-circumconic of the quadrangle.

The equation of this second circumconic is

$$P(q^2Q-r^2R)x^2+Q(r^2R-p^2P)y^2+R(p^2P-q^2Q)z^2=0$$

with the center $\left(\dfrac{1}{P(q^2Q-r^2R)}:\dfrac{1}{Q(r^2R-p^2P)}:\dfrac{1}{R(p^2P-q^2Q)}\right)$.



The center of the second conic is the *QA-Tr1*-tripole of the line connecting the center of the first circumconic with *QA-P16*.

Examples

***QA-Co2*: Nine-point Conic** (with e.g. incenter of *QA-Tr1*):

first center: *QA-P2*

second center:

$$\left(\frac{1}{(b^2r^2-c^2q^2)(2a^2q^2r^2-b^2r^2p^2-c^2p^2q^2} : \; ... \; \right)$$

***QA-Co3* Gergonne-Steiner Conic**

first center: *QA-P3*

second center: (too complex)

***QA-P1-QA-P16*-circumconic**

first center: $\left(\frac{p^2}{q^2-r^2} : \; ... \; \right)$

second center: $\left(\frac{p^2}{(q^2-r^2)(2p^2-q^2-r^2)} : \; ... \; \right)$

***QA-P5-QA-P10*-circumconic**

first center: $\left(\frac{1}{q^2-r^2} : \; ... \; \right)$

second center: $\left(\frac{1}{(q^2-r^2)(2p^2-q^2-r^2)} : \; ... \; \right)$

***QG-Co2* Circumscribed Harmonic Conic**

first center: *QG-P13*

second center: *QG-P1*

(the second conic degenerates in the diagonals)

***QG-Co3 M3D* Hyperbola**

first center: $\left(\frac{1}{q^2} : \frac{1}{r^2-p^2} : -\frac{1}{q^2}\right)$

second center: $\left(\frac{1}{p^2-2r^2} : \frac{q^2}{r^4-p^4} : \frac{1}{2p^2-r^2}\right)$

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