EQF-Note 2013-03-08

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A new Type of *EQF*-Transformation

The QA-Diagonal Triangle QA-Tr1 and the QL-Diagonal Triangle QL-Tr1 are reference triangles for isoconjugations. Here these transformations are upgraded. Then they show significant relationships in the QA-, QL- and QG-field. – Reference triangles for barycentric coordinates are QA-Tr1 and QL-Tr1.

1. Preliminary Notes

In the *QA*-field there is an isoconjugation wrt *QA-Tr1* with fixpoints in the vertices of the quadrangle (*QA-Tf2*):

$$(x:y:z) \quad\rightarrow\quad (p^2yz:q^2zx:r^2xy)\,.$$

This isoconjugation swaps some *QA*-points for example *QA-P10* and *QA-P16*.

In the *QL*-field there is no isoconjugation mentioned up to now. But we can define an isoconjugation wrt *QL-Tr1* with fixpoints in the trilinear poles of the sidelines of the quadrilateral:

$$(x:y:z) \quad\rightarrow\quad (m^2n^2yz:n^2l^2zx:l^2m^2xy)\,.$$

This isoconjugation swaps *QL-P8* and *QL-P13*.

These isoconjugations map lines into circumscribed conics of the reference triangle.

Combining two isoconjugations we get a new type of transformations. The second isoconjugation will be the isotomic conjugate wrt the reference triangle. These transformations aren´t involutary, but they map lines into lines.

In the *QA*-field:

$$QA-P16 \;\rightarrow\; QA-P10 \quad , \quad QA-P18 \;\rightarrow\; QA-P20\,,$$
$$QA-P1.QA-P16.QA-P21 \;\rightarrow\; QA-P10.QA-P16.QA-P19$$

In the *QL*-field: $\quad QL-P13 \;\rightarrow\; QL-P8\,,$

$$QL-P17 \;\rightarrow\; reflection\ of\ QL-P16\ in\ QL-P9\,,$$
$$QL-P13.QL-P17.QL-P24 \;\rightarrow\; QL-P1.QL-P8.QL-P24\,.$$

The relationships will grow up, if we take as third mapping the complement wrt the reference triangle.

2. The Transformations

Based on the preliminary notes we will consider the following chain of mappings in the *QA*- and *QL*-field.

isoconjugation → isotomic conjugate → complement.

The corresponding transformations shall be *QA-Tfx* and *QL-Tfx* with their inverse transformations *QA-Tfy* and *QL-Tfy*. The special difference to existing *EQF*-transformations will be:

***QA-Tfx,y* and *QL-Tfx,y* are not involutary transformations.**

QA-Tfx:

$$(x:y:z) \quad \rightarrow \quad (p^2(q^2z+r^2y):q^2(r^2x+p^2z):r^2(p^2y+q^2x))$$

QA-Tfy:

$$(x:y:z) \quad \rightarrow \quad (p^2(-x+y+z)):q^2(x-y+z):r^2(x+y-z))$$

QL-Tfx:

$$(x:y:z) \quad \rightarrow \quad (m^2y+n^2z:n^2z+l^2x:l^2x+m^2y)$$

*QL-Tf*y:

$$(x:y:z) \quad \rightarrow \quad (m^2n^2(-x+y+z)):n^2l^2(x-y+z):l^2m^2(x+y-z))$$

A significant property of these transformations is:

***QA-Tfx,y* and *QL-Tfx*,y map lines into lines.**

Additional remark: The background for

isotomic conjugate → complement

is the mapping of the Brianchon point to the center of an inscribed conic of the reference triangle. So *QL-Tfx* can be described in the following way: Consider the line pencil of a point and transform the lines with the “isoconjugation for lines”. The new lines will envelop an inscribed conic with center in the image of the chosen point.

3. Examples:

QA-Tfy* ←→*QA-Tfx	
lines	
line at infinity	***tripolar of QA-P20***
QA-L3	***QA-P1.QA-P16***
QA-P1.QA-P16	***QA-P10.QA-P16***
QL-L1	***QG-L2***
QG-L2	***QG-P1.QG-P2***
QG-L3	***QG-P2.QA-P16***
QG-P4.QG-P12	***QG-L3***
tripolar of QA-P16	***line at infinity***

QA-Tfy ←→QA-Tfx	
points	
QA-P1	*QA-P16*
QA-P16	*QA-P10*
QA-P18	*QA-P1*
QG-P1	*QG-P2*
QG-P12	*QG-P1*
QG-P13	*div. QG-P1.QG-P2 1:2*

QL-Tfy ←→QL-Tfx	
lines	
line at infinity	*QL-L9-parallel in QL-P19*
QL-L1	*line at infinity*
QL-L6	*QL-L2*
QL-L9	*QL-L1*
QL-P13.QL-P17	*mid-parallel QG-P1, QG-L1*
QG-L1	*QG-P1.QG-P2*
QG-L2	*QG-P1.QG-P3*
QG-L3	*QG-P13.QG-P15-parallel in QG-P3*
QG-P1.QG-P2	*QG-L1*
QG-P3.QG-P13	*QG-L2*
QG-P13.QG-P15	*QG-L3*

QL-Tfy ←→QL-Tfx	
points	
QL-P13	*QL-P8*
QL-P17	*QL-P1*
QL-P23	*point at infinity of QL-L1,4*
point at infinity of QL-L1,4	*point at infinity of QL-L9*
QL-L6 ∩ QL-L9	*QL-P7*
QL-L1∩QL-L6	*point at infinity of QL-L2,3*
point at infinity of QL-L2,3	*QL-P19*
QA-P1	*point at infinity of QG-P13.QG-P15*
QG-P2	*point at infinity of QG-L1*
QG-P3	*midpoint of QG-P1 and QG-L1 ∩ QG-L2*
QG-P4	*reflection of QG-P3 in QA-P1*

QG-P8	***reflection of QA-P1 in QG-P3***
QG-P12	***point at infinity of QG-P1.QG-P3***
QG-P13	***QG-P1***
QG-P15	***QA-P1***

4. Multiple Transformations and Fixpoints

QA-Tfx,y and *QL-Tfx,y* are not involutary transformations so they give sequences $\{P_i\}$ of points with the following properties:

P_i, P_{i+1}, P_{i+3} are collinear (*for QA-Tfx,y and QL-Tfx,y*).

The sequences $\{P_i\}$ are convergent in one of three fixpoints. These fixpoints depend on *QA-Tfx,y* and *QL-Tfx,y*.

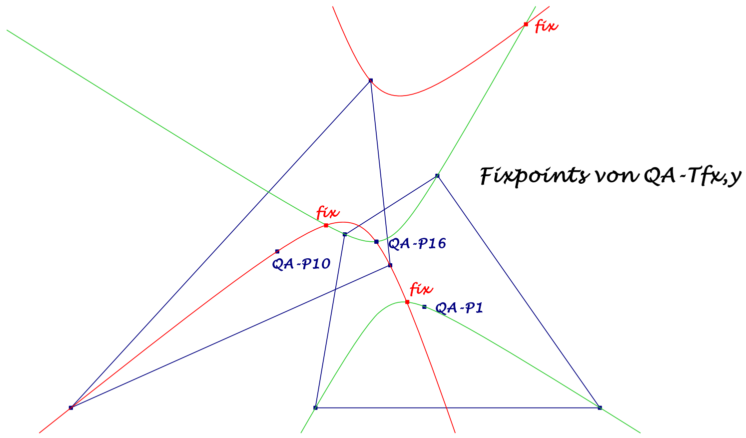
The corresponding properties hold for lines: limit line is one of the connecting lines of the fixpoints. The calculation of the fixpoints needs solutions of equations of degree 3, this shall not be treated. But there is a construction with two conics:

Fixpoints of *QA-Tfx,y*:

(1) Circumscribed conic of *QA-Tr1* through *QA-P10*, *QA-P16*, *QA-P18*. (This conic is the *QA-Tf2*-image of *QA-P10.QA-P16*).

(2) Circumscribed conic of the quadrangle through *QA-P1*, *QA-P16* (see *EQF*-Note *2013-01-18*).

Fixpoints are the intersections of both conics without *QA-P16*. Further properties of this triangle are described in *EQF*-Note *2013-02-01*.

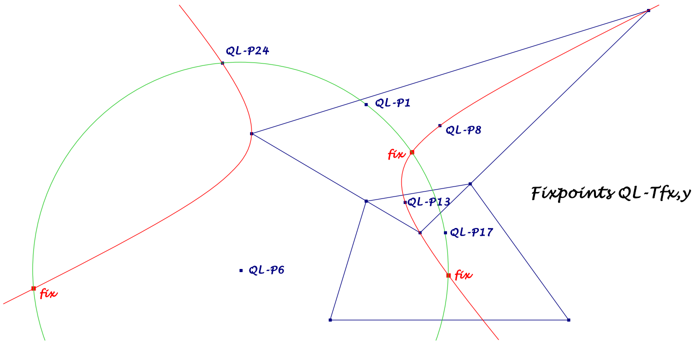


Fixpoints of *QL-Tfx,y*:

(1) Circumscribed conic of *QL-Tr1* through *QL-P8*, *QL-P13*, *QL-P24*. (This conic is the image of *QL-P8.QL-P13* for the isoconjugation in the *QL*-field).

(2) Dimidium Circle *QL-Ci6* .

Fixpoints are the intersections of both conics without *QL-P24*. For further properties of the fixpoints see *EQF*-Note *2013-02-01*.



5. Transformation of the Quadrangle / Quadrilateral

The vertices of the reference quadrangle can be transformed by *QA-Tfx,y* and the sidelines of the reference quadrilateral can be transformed by *QL-Tfx,y*.

QA-Tfx: **The image quadrangle is perspective to the reference quadrangle wrt *QA-P16*. The corresponding *QA*-diagonal triangle is the medium triangle of *QA-Tr1*.**

QA-Tfy: **The image quadrangle is perspective to the reference quadrangle wrt *QA-P1*. The corresponding *QA*-diagonal triangle is the anticevian triangle of *QA-P16* wrt *QA-Tr1*.**

QL-Tfx: **The image quadrilateral is line-perspective to the reference quadrangle wrt a parallel to *QL-L9* through *QL-P19*. The corresponding *QL*-diagonal triangle is the medium triangle of *QL-Tr1*.**

QL-Tfy: **The image quadrilateral is line-perspective to the reference quadrilateral wrt the line at infinity, that means: the sidelines are parallel to those of the reference quadrangle. The corresponding *QL*-diagonal triangle is the anticevian triangle of *QL-P13* wrt *QL-Tr1*.**

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