EQF-Note 2013-04-05

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Involutary Conjugate of Infinity Points

In EQF there are some QG-points Involutary Conjugates of infinity points of QG-lines (see tables for QA-Tf2). Here corresponding relationships are tested in the QA-environment.
The Involutary Conjugate QA-Tf2 is an isoconjugation wrt the QA-Diagonal Triangle QA-Tr1 with fixpoints in the vertices of the reference quadrangle. QA-Tf2 maps lines into circumscribed conics of QA-Tr1, especially the line at infinity in the Nine-point Conic QA-Co1. QA-Co1 is the locus of centers of circumscribed conics of the reference quadrangle. But in EQF there are only four circumscribed conics. Here the Involutary Conjugates of infinity points of QA-lines will be discussed wrt their collinearity with other QA-points. – Reference triangle for barycentric coordinates will be the QA-Diagonal Triangle QA-Tr1.

1. Circumscribed Conics of a Quadrangle

The Involutary Conjugate

$$(x:y:z) \quad \rightarrow \quad (p^2yz:q^2zx:r^2xy)$$

of infinity points are points on the Nine-point Conic *QA-Co1* with the equation

$$r^2xy+q^2zx+p^2yz=0$$

and the center

$$QA-P1=(p^2(-p^2+q^2+r^2):q^2(p^2-q^2+r^2):r^2(p^2+q^2-r^2)).$$

If we consider a line *L(e,f,g)*, the Involutary Conjugate of its infinity point

$$(\frac{p^2}{f-g}:\frac{q^2}{g-e}:\frac{r^2}{e-f})$$

will be a point of *QA-Co1* and a center of a circumscribed conic of the quadrangle. Up to now there are four circumscribed conics of a quadrangle in *EQF*:

***QA-Co2 QA*-Orthogonal Hyperbola**:
The center *QA-P2* is the Involutary Conjugate of infinity points of lines orthogonal to *QA-L2*.

_QA-Co3_ Gergonne-Steiner Conic:
The center _QA-P3_ is the Involutary Conjugate
of the infinity point of _QA-L4_.

_QA-2Co1a,b_ Pair of Circumscribed _QA_-Parabolas:
The axes of the _QA_-Parabolas are parallel to the asymptotes of _QA-Co1_; their infinity points are Involutary Conjugates.

There are two further circumscribed conics of a quadrangle (see my EQF-Note 2013-01-18):

Circumscribed conic through _QA-P5_ and _QA-P10_:
The center, reflected in _QA-P1_, is the Involutary Conjugate
of the infinity point of _QA-P1.QA-P16_.

Circumscribed conic through _QA-P1_ and _QA-P16_:
The center, reflected in _QA-P1_, is the Involutary Conjugate
of the infinity point of _QA-L3_.

2. Special Involutary Conjugates of Infinity Points

Considering *QA*-lines, the Involutary Conjugates of their infinity points on *QA-Co1* are sometimes collinear with *QA-P2* and another *QA*-point. For example: The Involutary Conjugate of the infinity point of *QA-P4.QA-P8* is the second intersection point of *QA-P2.QA-P7* and *QA-Co1*.

infinity point of line …	***Involutary Conjugate***
QA-L4	***QA-Co1 ^ QA-P2.QA-P3***
QA-P1.QA-P11	***QA-Co1 ^ QA-P2.QA-P37***
QA-P1.QA-P28	***QA-Co1 ^ QA-P2.QA-P15***
QA-L2	***QA-Co1 ^ QA-P2.QA-P23***
QA-P2.QA-P23	***QA-Co1 ^ QA-P2.QA-P6***
QA-P4.QA-P8	***QA-Co1 ^ QA-P2.QA-P7***
QA-P4.QA-P12	***QA-Co1 ^ QA-P2.QA-P11***
QA-P10.QA-P14	***QA-Co1 ^ QA-P2.QA-P14***
QA-P11.QA-P23	***QA-Co1 ^ QA-P2.QA-P36***

If the Involutary Conjugate of an infinity point is reflected in *QA-P1* – always a point on *QA-Co1* – there are corresponding properties wrt *QA-P3*.

infinity point of line …	***Involutary Conjugate reflected in QA-P1***
QA-L1	***QA-Co1 ^ QA-P3.QA-P4***
QA-L2	***QA-Co1 ^ QA-P3.QA-P32***
QA-L4	***QA-Co1 ^ QA-P3.QA-P2***
QA-P1.QA-P4	***QA-Co1 ^ QA-P3.QA-P8***
QA-P1.QA-P11	***QA-Co1 ^ QA-P3.QA-P12***
QA-P1.QA-P12	***QA-Co1 ^ QA-P3.QA-P30***

In *QG*-environment there are some further results: As mentioned above the first three are to be found in *EQF* under *QA-Tf2*.

infinity point of line …	***Involutary Conjugate***
QG-P1.QG-P2	***QG-P13***
QG-P1.QG-P3	***QG-P14***
QL-L1	***QG-P15***
QG-L2	***QA-Co1 ^ QG-P1.QG-P2***
QG-L3	***refl. of QG-P14 in QA-P1***

3. Infinity Points of orthogonal Lines

All orthogonal hyperbolas circumscribed *QA-Tr1* contain the orthocenter *QA-P12*. The Involutary Conjugate of *QA-P12* is the Inscribed Square Axes Crosspoint *QA-P23*. Therefore *QA-P23* can be considered as the common point of all *QA-Tf2*-images of orthogonal hyperbolas circumscribed *QA-Tr1*.

For orthogonal lines the Involutary Conjugates of their infinity points lie on *QA-Co1* with a chord through *QA-P23*.

Remark: Consider chords of *QA-Co1* through *QA-P23*. The Thales-circle over a chord intersects *QA-Co1* in *QA-P2* and a further point *X*. The orthogonal hyperbola circumscribed *QA-Tr1* through *X* cuts the chord in points of an isocubic wrt *QA-Tr1* for pivot *QA-P23* and isoconjugation *QA-Tf2*.

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