EQF-Note 2013-04-12

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

QL-P5, QL-P12, QL-P20, QL-P22

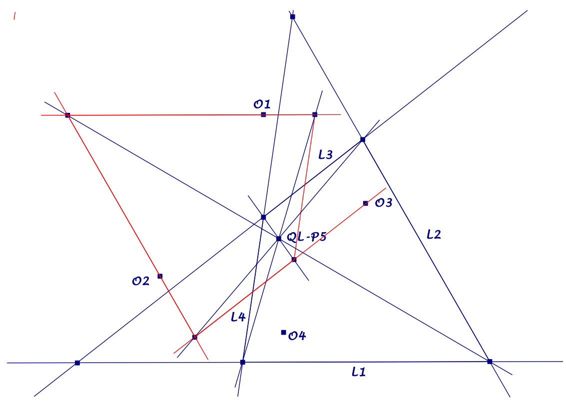
For quadrilaterals with the same Inscribed Parabola QL-Co1 and the same QL-Diagonal Triangle QL-Tr1 the loci for the points above are lines with a common point. This is the reason for the following research. – Reference triangle for barycentric calculation is QL-Tr1.

The Points

The four points above lie collinear on the Newton line *QL-L1* with rational distance ratios:

$$P5.P12 : P12.P22 : P22.P20 = 2:1:3 .$$

These are the ratios of the Euler line. Background: Take the Euler lines of the triangle components, choose on each Euler line the same special point *X* and draw a parallel to the corresponding sideline of the quadrilateral, then the constructed quadrilateral is homothetic to the reference quadrilateral with center *Y*.



X	Y	
circumcenter	*QL-P5*	*(see figure)*
centroid	*QL-P12*	*(see EQF)*
nine-point center	*QL-P22*	*(see EQF)*
orthocenter	*QL-P20*	*(see EQF)*

The Lines

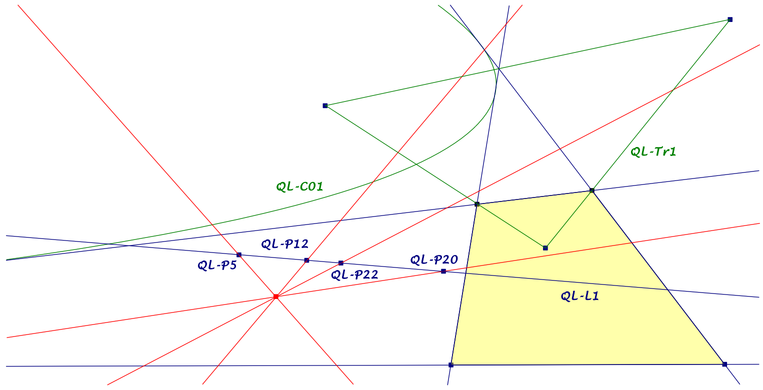
We consider the pencil of quadrilaterals with the same *QL-Diagonal Triangle QL-Tr1* and the same Inscribed Parabola *QL-Co1*. The inscribed parabola has the equation:

$$\frac{x^2}{m^2-n^2}+\frac{y^2}{n^2-l^2}+\frac{z^2}{l^2-m^2}=0\,.$$

For a shorter description the point at infinity of the parabola shall be

$$(u:v:w)=(m^2-n^2:n^2-l^2:l^2-m^2) \quad with \quad u+v+w=0\,.$$

With $l=\sqrt{m^2+w}$ *and* $n=\sqrt{m^2-u}$ we have a parameter m for the coefficients of the defining lines of the quadrilateral.



The loci of the considered points are lines with the equations:

QL-P5: $\sum_{cycl}(a^4vw+b^4wu+c^4uv+4a^2S_Avw)x=0$

This line through *QL-P5* is orthogonal to *QL-P1.QL-P9*.

QL-P12: $\sum_{cycl}vwx=0$

This line is the polar of *QL-P8* wrt the parabola.

QL-P20: $\sum_{cycl}(b^2w+c^2v)^2x=0$

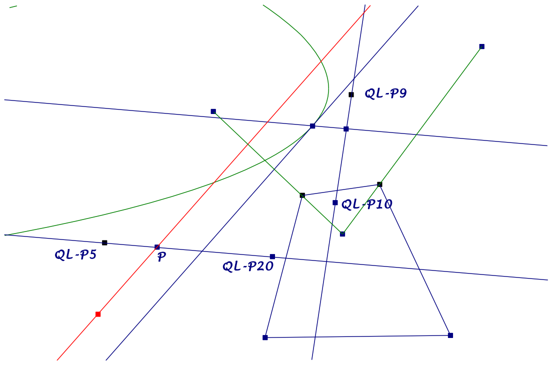
Construction of this line: a parallel to *QL-L1* through *QL-P10* (*QL-DT*-orthocenter), intersection with the parabola, tangent at the parabola, a parallel through *QL-P20*.

QL-P22: $\sum_{cycl}(4S^2vw+(-S_Au+S_Bv+S_Cw)^2)x=0$

Construction of this line: a parallel to *QL-L1* through *QL-P11* (*QL-DT*-nine-point center), intersection with the parabola, tangent at the parabola, a parallel through *QL-P22*.

Within the meaning of the *X-Y*-list above the last two constructions can be generalized:

The locus of a point P on *QL-L1*, dividing *QL-P5.QL-P20* in the ratio κ, can be constructed as follows: a parallel to *QL-L1* through a point, dividing *QL-P9.QL-P10* in the same ratio κ, intersection with the parabola, tangent at the parabola, a parallel through *P*.



Equation of this line:

$$\sum_{cycl}(-a^4vw+b^4wu+c^4uv+2a^2(b^2+c^2)vw+2(c^2v+b^2w)^2\kappa)x)=0$$

All these lines have a common point.

For quadrilaterals with the same inscribed parabola and the same *QL*-diagonal triangle the loci for the points *QL-P5, QL-P12, QL-P20, QL-P22* are lines with a common point.

This common point has the coordinates:

$$(u(v(a^2w+c^2u)^2-w(a^2v+b^2u)^2)$$
$$:(v(w(b^2u+a^2v)^2-u(b^2w+c^2v)^2)$$
$$:(w(u(c^2v+a^2w)^2-v(c^2u+a^2w)^2)).$$

The polar of this point wrt the inscribed parabola is a line through *QL-P8*. The direction of the polar is orthogonal to a line through *QL-P10* and a point, which is the intersection of *QL-L2* and a perpendicular line to *QL-L7* through *QL-P3*.

– Unfortunately no further properties can be given. –

A special Case

Taking three lines for a reference triangle and a fourth line parallel to the Euler line of the reference triangle, then we get a quadrilateral, where the four considered points coincide.

For quadrilaterals with sidelines parallel to the Euler lines of their triangle components holds *QL-P5 = QL-P12 = QL-P20 = QL-P22.*

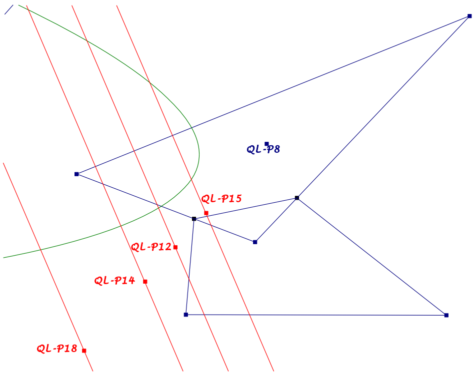
There are four quadrilaterals of this type among those with the same inscribed parabola and the same *QL*-diagonal triangle, taking lines

$$(\pm\sqrt{3c^4v^2+2(3S_A^2+S^2)vw+3b^4w^2},$$
$$\pm\sqrt{3a^4w^2+2(3S_B^2+S^2)wu+3c^4u^2},$$
$$\pm\sqrt{3b^4u^2+2(3S_C^2+S^2)uv+3a^4v^2}).$$

for defining the quadrilateral.

Final Remark

Among the quadrilaterals of the considered pencil there are four further collinear points, whose loci are lines with a common point. The common point is a point at infinity, so the lines are parallel. The first one is already mentioned.



QL-P12: $\sum_{cycl} vwx = 0$

This line is the polar of *QL-P8* wrt the parabola.
Polar distance from *QL-P8:*

$$d=\frac{S(uv+vw+wu)}{3\sqrt{S_Au^2(v-w)^2+S_Bv^2(w-u)^2+S_Cw^2(u-v)^2}}$$

QL-P14: $\sum_{cycl}(u^2-10vw)x=0$

Parallel line with distance 4/3d from *QL-P8*.

QL-P15: $\sum_{cycl}(u^2+8vw)x=0$

Parallel line with distance 2/3d from *QL-P8*.

QL-P18: $\sum_{cycl}(v-w)^2x=0$

Parallel line with distance 2d from *QL-P8*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de