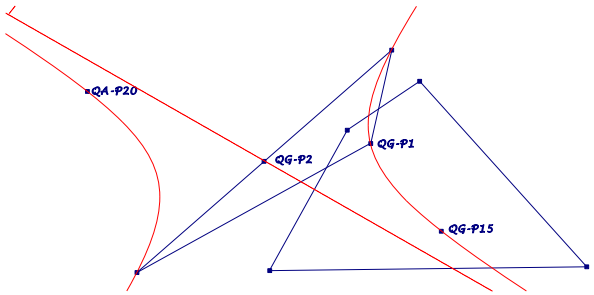
EQF-Note 2013-04-15

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Involutary Conjugate of the Newton Line

For a quadrigon a QA-DT circumscribed conic with center QG-P2 through QG-P15 will be discussed. – Reference triangle for barycentric coordinates is QA-DT.



The Conic in *QG*-environment

In the *QG*-environment we consider the Newton Line

$$QL\text{-}L1: \qquad q^2x+(p^2-r^2)y-q^2z=0\,.$$

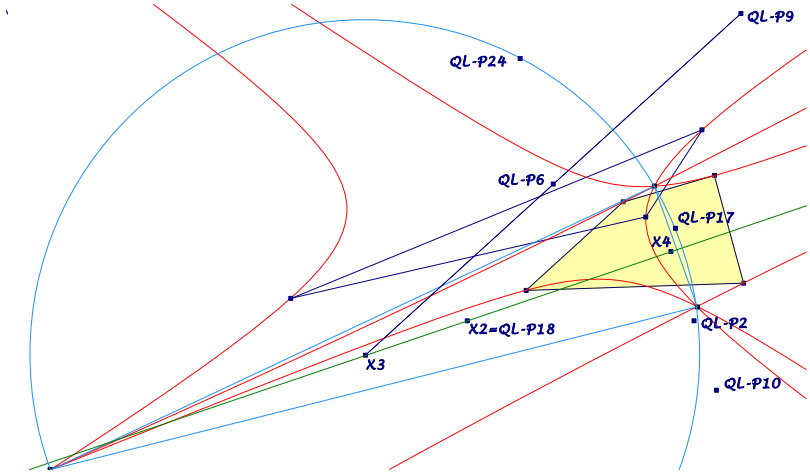
Its Involutary Conjugate (*QA-Tf2*) is a *QA-DT* circumscribed conic with the equation

$$-r^2xy+(p^2-r^2)zx+p^2yz=0\,.$$

- The center of the conic is *QG-P2*.
- The conic contains the points
  QG-P15 (image of the point at infinity of *QL-L1*),
  QA-P20 (image of *QA-P1*),
  reflection of *QG-P1* in *QG-P2* (image of *QG-P12*),
  the reflections of *QG-P15*, *QA-P20* in *QG-P2*.
- The asymptotes are parallel to the legs of the *QL*-diagonal triangle. Their points at infinity are the images of the midpoints of the diagonals of the quadrigon.
- The tangent in *QG-P1* is *QG-L2*,
  the tangent in *QG-P15* is *QG-P12.QG-P15*,
  the pole of *QG-L3* is *QG-P12*.
  The tangents in the endpoints of the 3rd diagonal are parallel *QG-P1.QG-P3*.
- The conic is the locus of *QG-P15* for all quadrigons with the same *QA*- and *QL*-diagonal triangle.
- The conic divides the sidelines in ratios with product 1.

The Conic in *QL*-environment

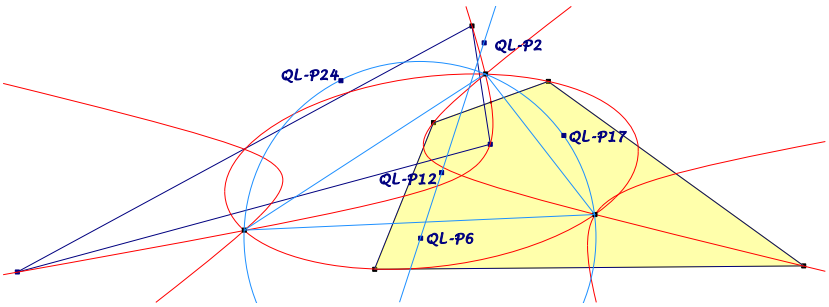
For a quadrangle there are three conics of the new type. Common points are the vertices of *QA-DT* and *QA-P20*. For a quadrilateral there are also three conics but with three common points. The calculation needs solutions of equations with degree 3. So the following interesting properties are only Cabri controlled.



- The three conics have three common points.
- The centroid *X2* of this triangle is *QL-P18* (reflection of *QL-P8* in *QL-P12*).
- The orthocenter *X4* of this triangle is the reflection of *QL-P10* in *QL-P2*.
- The circumcenter *X3* of this triangle is the reflection of *QL-P9* in *QL-P6*.
- The circumcircle of this triangle contains *QL-P17* and *QL-P24*.
- The Simson line of *QL-P17* wrt this triangle is a parallel to *QL-L6* half the distance to *QL-P17*.

Comparison with the Nine-point Conic *QA-Co1*

We can compare these properties with those of another conic. Taking the three Nine-Point Conics *QA-Co1* for a quadrilateral, we get also three common points (see *QL-P6* in *EQF*).



The following list shows centroid, circumcenter and orthocenter of the *QL*-diagonal triangle *QL-DT*, the triangle wrt *QA-Co1* and the triangle wrt *QG-Cox*.

	QL-DT	*QA-Co1-*Δ	*QG-Cox-*Δ
centroid	*QL-P8*	*QL-P12*	*QL-P18*
circumcenter	*QL-P9*	*QL-P6*	*see above*
orthocenter	*QL-P10*	*QL-P2*	*see above*

Reflecting one of the *QL-DT*-points in the corresponding point of *QA-Co1*-Δ we get the corresponding point of *QG-Cox*-Δ. The circumcircles of the three triangles contain *QL-P17* and *QL-P24*.

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