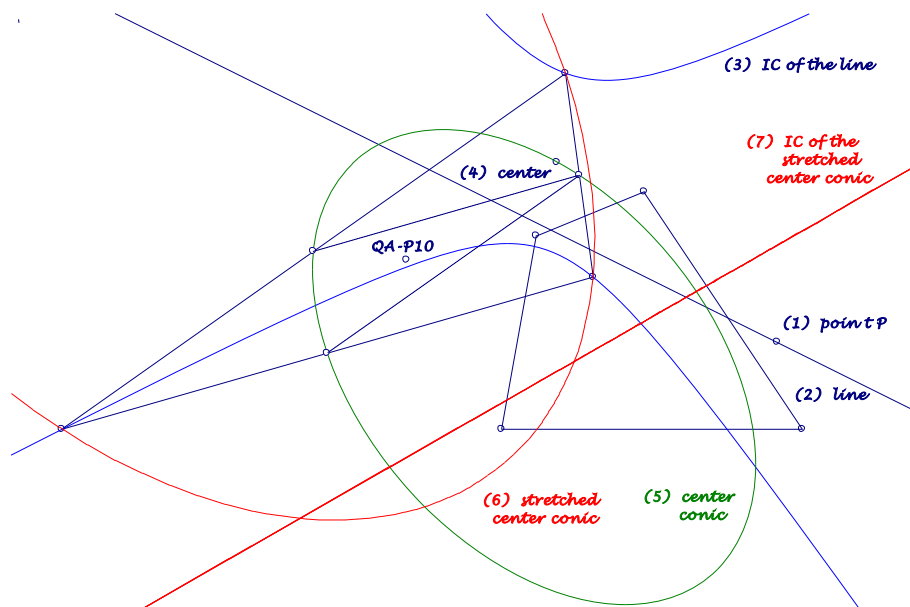


Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Circumscribed Conics of the QA-DT Medial Triangle

A special construction leads from an arbitrary point P to a circumscribed conic of the QA-DT medial triangle. Stretching this conic to a circumscribed conic of QA-DT it is the Involutory Conjugate of a line L . Here several examples are listed without right to completeness. – Reference triangle for barycentric coordinates is QA-DT.



Construction

- Line pencil of an arbitrary point $P(u:v:w)$.
- Involutory conjugates of the lines are QA-DT circumscribed conics.
- Locus of the centers is a circumscribed conic of the medial triangle of QA-DT:

$$q^2r^2ux^2 + r^2p^2vy^2 + p^2q^2wz^2 - p^2(q^2w + r^2v)yz - q^2(r^2u + p^2w)zx - r^2(p^2v + q^2u)xy = 0$$

with center

$$(2p^2vw + q^2wu + r^2uv : p^2vw + 2q^2wu + r^2uv : p^2vw + q^2wu + 2r^2uv).$$

- The anticomplement of the center conic is a circumscribed conic of $QA-DT$:

$$p^2(q^2w + r^2v)yz + q^2(r^2u + p^2w)zx + r^2(p^2v + q^2u)xy = 0$$

with center
 $(u(q^2w + r^2v) : v(r^2u + p^2w) : w(p^2v + q^2u))$.

- The Involutary Conjugate of the stretched center conic is a line:

$$(q^2w + r^2v)x + (r^2u + p^2w)y + (p^2v + q^2u)z = 0.$$

Examples

$P = QA-P1$

The center conic contains $QA-P29$ and has center $QA-P22$.

The stretched center conic is the Nine-point Conic $QA-Co1$.

The final line is the line at infinity.

$P = QA-P2$

The center conic and the stretched center conic are parabolas with axis orthogonal to $QA-L2$.

The final line contains $QA-P2$ and is orthogonal to $QA-P2.QA-P23$.

$P = QA-P3$

The center conic and the stretched center conic are parabolas with axis parallel to $QA-L4$.

The final line contains $QA-P3$ and is orthogonal to $QA-P3.QA-P32$.

$P = QA-P6$

The center conic is an orthogonal hyperbola through $QA-P11$ and $QA-P29$ with center in the midpoint of $QA-P11.QA-P29$.

The stretched center conic is an orthogonal hyperbola through $QA-P12$ and has the center $QA-P36$.

The final line contains $QA-P23$ and is orthogonal to $QA-P1.QA-P32$.

$P = QA-P10$

The center conic has a center in the midpoint of $QA-P16.QA-P31$.

The center of the stretched center conic is $QA-P31$.

The final line is the $QA-DT$ trilinear polar of the isotomic conjugate of $QA-P31$ (orthogonal to $QA-P12.QA-P32$).

P = QA-P16

The center conic is the *QA-DT* inscribed Steiner ellipse with center *QA-P10*.

The stretched center conic is the *QA-DT* circumscribed Steiner ellipse with center *QA-P10*.

The final line is the *QA-DT* trilinear polar of *QA-P16*.

QA-P17

The center of the center conic is *QA-P1*.

The center of the stretched center conic is *QA-P20*.

The final line is a *QA-DT* trilinear polar of a point on *QA-P1.QA-P17...*

QA-P18

The center of the center conic is *QA-P31*.

The center of the stretched center conic is *QA-P16*.

The final line is the trilinear polar of *QA-P20*.

QA-P19

The center of the center conic divides *QA-P10.QA-P18* with ratio *1:3*.

The center of the stretched center conic divides *QA-P10.QA-P18* with ratio *-1:3*.

QA-P20

The center of the center conic is the midpoint of *QA-P1.QA-P22*.

The center of the stretched center conic is *QA-P22*.

The final line is orthogonal to *QA-P12.QA-32...*

QA-P23

The center conic is the circumcircle *QA-Ci2* of the *QA-DT* medial triangle.

The stretched center conic is the circumcircle *QA-Ci1* of *QA-DT*.

The final line is the connection of *QA-P6* and the reflection of *QA-P38* in *QA-Ci1* (orthogonal to *QA-P1.QA-P32* and *QA-P11.QA-P38*). It is the *QA-DT* trilinear polar of a point which is the isotomic conjugate of the isogonal conjugate of *QA-P16*:

$$a^2q^2r^2x + b^2r^2p^2y + c^2p^2q^2z = 0 .$$

QA-P27

The center of the center conic is the midpoint of $QA-P1.QA-P31$ (or divides $QA-P10.QA-P21$ with ratio $1:3$).
The center of the stretched center conic divides $QA-P10.QA-P21$ with ratio $-1:3$.

QA-P30

The center of the center conic divides $QA-P10.QA-P6$ with ratio $1:3$.
The center of the stretched center conic divides $QA-P10.QA-P6$ with ratio $-1:3$.

QA-P36

The final line is orthogonal to $QA-P12.QA-P29$.

In the QG -environment the center conic always contains $QG-P2$ and the stretched center conic always contains $QG-P1$ (further not mentioned). The parallel to $QG-L1$ half the distance to $QG-P1$ shall be denoted as QG -mid-parallel.

QG-P1

The construction degenerates.

QG-P2

The center of the center conic is the midpoint of $QG-P1.QG-P2$. The center conic contains $QG-P1$ and the intersection of $QG-L1$ and $QG-L2$.
The stretched center conic has center $QG-P2$.
The final line is a parallel to $QG-L1$ through $QG-P12$.

QG-P3

The center conic degenerates into two lines: $QG-P1.QG-P2$ and QG -mid-parallel.

QG-P4

The center of the stretched center conic lies on $QG-P2.QG-P12$. It is the reflection of the intersection of $QG-P2.QG-P12$ and the QG -mid-parallel in $QA-P1$.
The final line is parallel to $QG-P1.QG-P3$ through the reflection of $QG-P1$ in $QG-P15$.

QG-P12

The Involutary Conjugate of all lines through *QG-P12* are *QA-DT* circumscribed conics with center *QG-P2*. So the construction degenerates.

QG-P13

The center conic is a parabola with an axis parallel to *QG-P1.QG-P2* and *QG-L1* tangent in *QG-P2*.

The stretched center conic is a parabola with an axis parallel to *QG-P1.QG-P2* and *QG-P1.QG-P14* tangent in *QG-P1*.

The final line is *QG-P3.QG-P13*.

QG-P14

The center conic is a parabola through *QG-P3* with an axis parallel to *QG-P1.QG-P3*.

The stretched center conic is a parabola through the *QA-DT*-anticomplement of *QG-P3* with an axis parallel to *QG-P1.QG-P3*.

The final line contains *QG-P14*.

QG-P15

The center conic is a parabola with an axis parallel to *QG-P2.QG-P12*.

The stretched center conic is a parabola through *QG-P14* with an axis parallel to *QG-P2.QG-P12*.

The final line is parallel to *QG-P1.QG-P3* through *QG-P15*.

Eckart Schmidt

<http://eckartschmidt.de>

eckart_schmidt@t-online.de