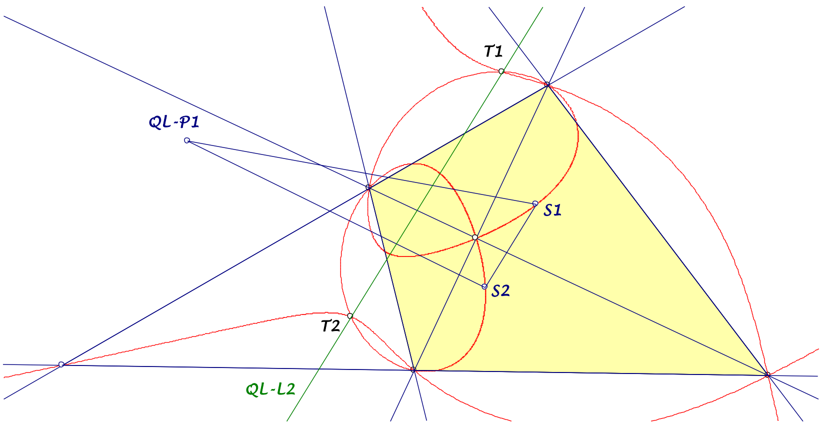
EQF-Note 2013-10-06

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Pair of QG-Cubics

The loci for points, which have the same angle of view (mod 180°) for the diagonals of a quadrigon, are two cubics. – Reference triangle for barycentric coordinates is QA-Diagonal Triangle QA-Tr1.



Constructing about and under the diagonals of a quadrigon the circles for the same peripheral angle, there are four points of intersection, which have as loci two cubics with the equations

$$2S_A y(r^2x^2 - p^2z^2) - 2S_B x(r^2y^2 - q^2z^2)$$
$$+(c^2q^2 + b^2r^2)x^2z - (c^2p^2 + a^2r^2)y^2z - (b^2p^2 - a^2q^2)z^3 = 0,$$
$$2S_B y(q^2x^2 - p^2y^2) - 2S_C y(r^2x^2 - p^2z^2)$$
$$+(b^2p^2 + a^2q^2)xz^2 - (c^2p^2 + a^2r^2)xy^2 - (b^2r^2 - c^2q^2)x^3 = 0.$$

The points at infinity are

$$(p^2 - q^2 + r^2 : -p^2 + q^2 + r^2 : -2r^2)$$
$$(-2p^2 : p^2 + q^2 - r^2 : p^2 - q^2 + r^2).$$

From the points of these cubics the diagonals of the quadrigon are seen under angles mod 180°.

Properties:

- Both cubics are circumcubics of the quadrigon.
- Both cubics contain the diagonal crosspoint *QG-P1*.
- Each cubic contains one of the intersections *QG-2P2a,b* of opposite sides.
- Further intersections of the two cubics are the intersections *T1*, *T2* of the Thales circles about the diagonals.
- The line *T1T2* is *QL-L2*.
- Each of the cubics contains one of the vertices *S1*, *S2* of the Miquel triangle *QA-Tr2* unequal *QL-P1*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de