EQF-Note 2013-10-07

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

QL-DT Circumscribed Conics

For a triangle circumscribed conics are isoconjugates of lines. In the QA-environment circumscribed conics of QA-DT are the Involutary Conjugates (QA-Tf2) of lines, for this transformation is an isoconjugation wrt QA-DT with fixed points in the vertices of the quadrangle. Examples: QA-Co1, QA-Co4, QA-Co5. In the QL-environment no circumscribed conics of QL-DT (without the circumcircle) are mentioned in EQF. Here are given some examples. – Reference triangle for barycentric coordinates is QL-DT.

QL-Isoconjugation

Wrt *QL-DT* there is an isoconjugation with fixed points in the trilinear poles of the side lines of the quadrilateral:

$$(-mn:nl:lm),\ (mn:-nl:lm),\ (mn:nl:-lm),\ (mn:nl:lm).$$

This *QL*-isoconjugation (for points) is the mapping

$$(x:y:z) \quad\rightarrow\quad (m^2n^2yz:n^2l^2zx:l^2m^2xy)$$

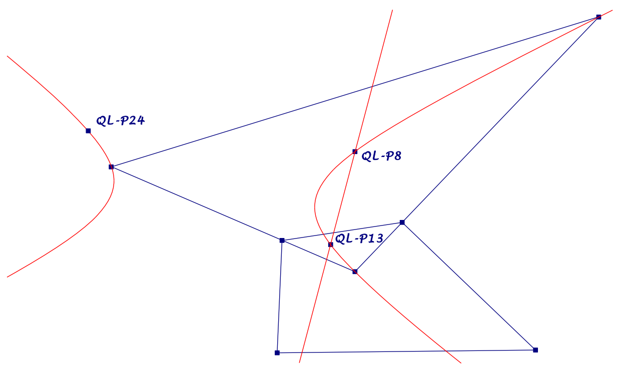
with the special example *QL-P8* $\rightarrow$ *QL-P13*.

Lines will be mapped in *QL*-circumscribed conics, but there are only a few interesting examples.

QL-L1 The *QL-DT* circumscribed conic is the Steiner ellipse.

QL-L7,8 The *QL-DT* circumscribed conics contain *QL-P13*.

QL-P8.QL-P13 The *QL-DT* circumscribed conic contains beside *QL-P8* and *QL-P13* further *QL-P24*.



Another Way to *QL-DT* Circumscribed Conics

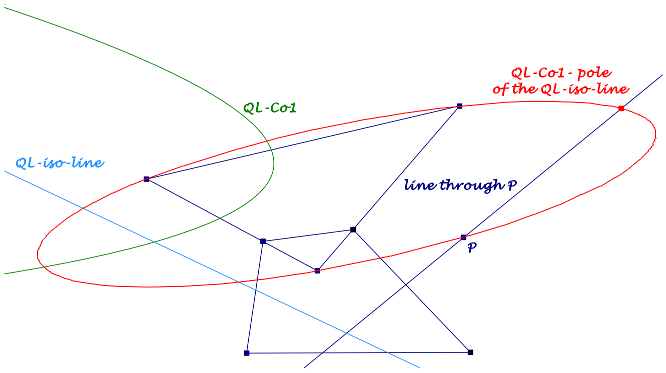
Wrt *QL-DT* there is an isoconjugation for lines (*QL-Tf2*). Fixed lines are the side lines of the quadrilateral. This *QL*-isoconjugation for lines is the mapping

$$(e, f, g) \rightarrow (l^2 fg, m^2 ge, n^2 ef).$$

You get the image line taking the trilinear pole, its *QL*-isoconjugate (for points) and then the trilinear polar.

Construction of a special *QL-DT* circumscribed conic through an arbitrary point *P(u:v:w)*:
Consider the line pencil of *P* and the images of the lines wrt the *QL*-isoconjugation for lines. The poles wrt the Inscribed Parabola *QL-Co1* are points on the lines again. The locus of these points is a *QL-DT* circumscribed conic with the equation

$$(l^2 - m^2)n^2 w\, xy + (m^2 - n^2)l^2 u\, yz + (n^2 - l^2)m^2 v\, zx = 0.$$



Some Examples:

P* = *QL-P2 The *QL-DT* circumscribed conic is an orthogonal hyperbola through *QL-P2* and *QL-P10*.

P* = *QL-P7 The *QL-DT* circumscribed conic contains *QL-P7* and the point at infinity of *QL-L1* (one asymptote parallel to *QL-L1*).

P* = *QL-P10 The *QL-DT* circumscribed conic is an orthogonal hyperbola through *QL-P10*.

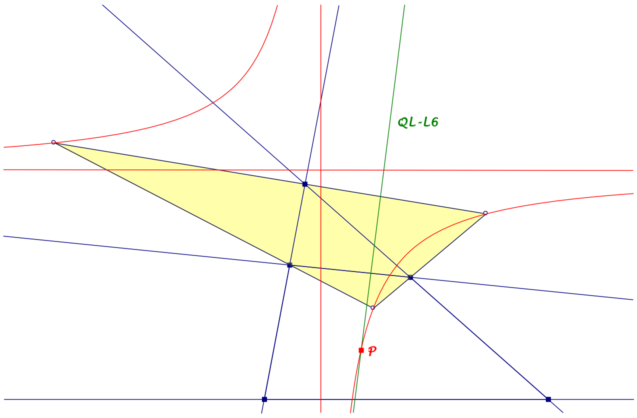
P* = *QL-P13 The *QL-DT* circumscribed conic contains *QL-P8, QL-P13, QL-P24* (see example above).

P* = *QL-P17 The *QL-DT* circumscribed conic is the circumcircle *QL-Ci1*.

P* = *QL-P23 The *QL-DT* circumscribed conic is a parabola with an axis parallel *QL-L1*.

All points on the line *QL-L6* give *QL-DT* circumscribed orthogonal hyperbolas.

Example: The point at infinity gives an orthogonal hyperbola through *QL-P10* with asymptotes parallel to *QL-L6* and *QL-L9*.

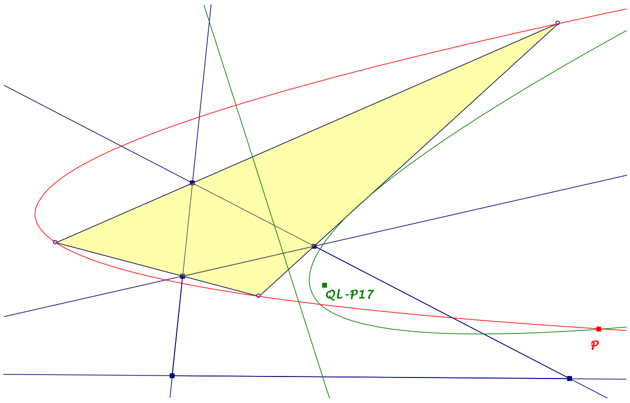


All points on the parabola with focus *QL-P17* and directrix *QL-L6* give *QL*-circumscribed parabolas.

The reference parabola has the equation

$$\sum_{cycl}(l^4(m^2-n^2)^2x^2-2(l^2-m^2)(l^2-n^2)m^2n^2yz)=0.$$

It is an inscribed conic of *QL-DT*. *QL-L1* is tangent to this parabola in *QL-P23*. Mapping the parallels to *QL-L1* by the *QL*-isoconjugation for lines (see above), the envelope gives this parabola.



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