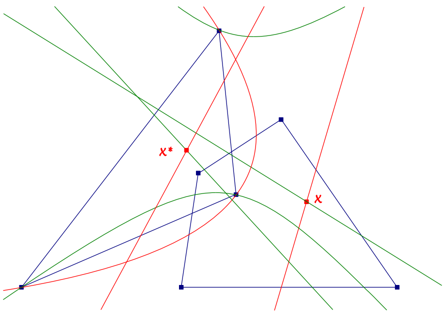
EQF-Note 2013-10-13

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Curious *QA*-Transformation

This is a new type of QA-transformation: not an involution but holding fixed the vertices of the quadrangle as well as the six connection lines. Further properties are discussed. – Reference triangle for barycentric coordinates is the QA-Diagonal Triangle QA-DT.



Definition *QA-Tfx*:

Let *X* be a point, consider lines through *X* and their Involutary Conjugates (*QA-Tf2*), which are *QA-DT* - circumconics. The polars of *X* wrt these conics have a common point *X.**

For $X=(x:y:z)$ the image point is

$$X^*=(x(-q^2r^2x^2+r^2p^2y^2+p^2q^2z^2) : y(q^2r^2x^2-r^2p^2y^2+p^2q^2z^2) : z(q^2r^2x^2+r^2p^2y^2-p^2q^2z^2)).$$

Properties:

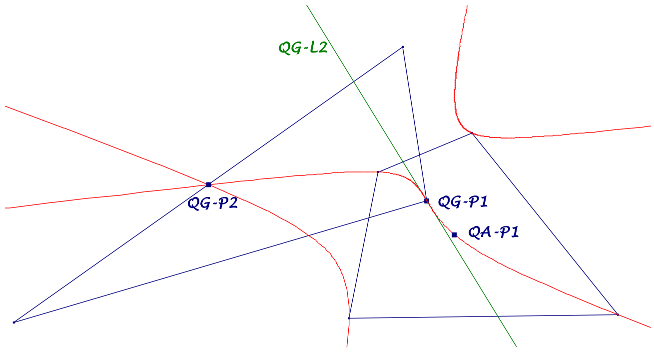
1. *QA-Tfx* is not an involution.
2. The vertices P_i of the quadrangle as well as the vertices S_i of *QA-DT* are fixed points.
3. The lines P_iP_j of the quadrangle as well as the sidelines S_iS_j of *QA-DT* are invariant wrt *QA-Tfx*.
4. All points on the trilinear polar of P_i wrt *QA-DT* have the image P_i.
5. All points *X*, *X** on a sideline S_iS_j of *QA-DT* divide $S_i.S_j$ harmonic.

6. Circumscribed conics of the quadrangle are invariant under *QA-Tfx*.
7. *QA-Tfx* mappings of lines unequal P_iP_j and S_iS_j are circumcubics of the quadrangle. For a line with the coefficients *e*, *f*, *g* The cubic has the equation

$$eq^2r^2(f^2q^2-g^2r^2)x^3+fr^2p^2(g^2r^2-e^2p^2)y^3+gp^2q^2(e^2p^2-f^2q^2)z^3$$
$$+fq^2r^2(2e^2p^2-f^2q^2+g^2r^2)x^2y+ep^2r^2(e^2p^2-2f^2q^2-g^2r^2)xy^2$$
$$+gq^2r^2(-2e^2p^2-f^2q^2+g^2r^2)x^2z+ep^2q^2(e^2p^2-f^2q^2-2g^2r^2)xz^2$$
$$+gp^2r^2(e^2p^2+2f^2q^2-g^2r^2)y^2z+fp^2q^2(-e^2p^2+f^2q^2-2g^2r^2)yz^2=0$$

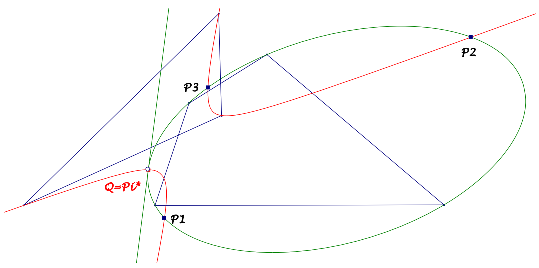
Example *QA-Tfx(QG-L2)* with equation:

$$(p^2-r^2)(p^2z-r^2x)y^2-q^2(z-x)^2(p^2z+r^2x)=0$$



This circumscribed cubic of the quadrangle contains *QG-P1* as inflexion point, *QG-P2* as nodal point and *QA-P1*.

8. For a point *Q* on a circumconic of a quadrangle there are three points *Pi* with *QA-Tfx*-image *Q*. All four points lie on a circumconic of *QA-DT*, which is the *QA-Tf2* image of the tangent at *Q* to the *QA*-circumconic.



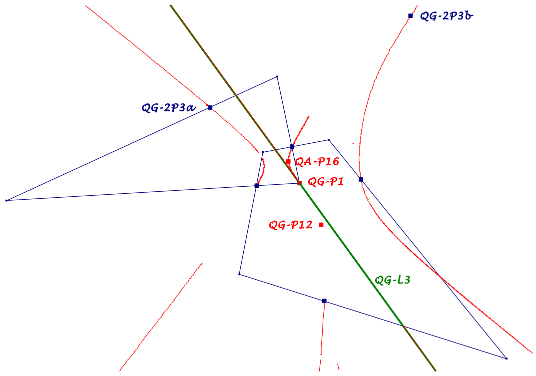
9. For a line *L* unequal P_iP_j and S_iS_j there is a cubic, whose *QA-Tfx*-image is the line *L*.
   If the line has the coefficients *e*, *f*, *g*, the equation of the cubic is

$$-eq^2r^2x^3-fr^2p^2y^3-gp^2q^2z^3+fq^2r^2x^2y+er^2p^2xy^2$$
$$+gp^2r^2y^2z+fp^2q^2yz^2+ep^2q^2xz^2+fp^2q^2yz^2=0\,.$$

   The cubic can be constructed in the following way: Let *X* be a point of the line *L* and Co_1 the *QA*-circumconic through *X*. The *QA-Tf2*-image of the tangent in *X* at Co_1 is a *QA-DT*-circumconic Co_2. The intersections of Co_1 and *Co2* are points of the cubic.

Example: *QG*-Cubic with *QA-Tfx*-image *QG-L3* and equation

$$p^2(-p^2+q^2+r^2)(q^2r^2x^2+r^2p^2y^2-p^2q^2z^2)z$$
$$+r^2(p^2+q^2-r^2)(-q^2r^2x^2+r^2p^2y^2+p^2q^2z^2)x=0$$



This cubic contains *QG-P1* as inflexion point, *QG-2P3a,b*, *QA-P16* and *QG-P12*. Further points on the cubic are the intersections of the lines *QG-P1.QG-2P2a,b* with the sidelines of the quadrigon.

Final remark

There are analogous relationships in the *QL*-environment, defining the following transformation for lines:

Definition *QL-Tfx*:

Let *L* be a line, consider points *X* on *L* and for the lines through *X* the *QL*-Line Isoconjugates (*QL-Tf2*), which envelope *QL-DT*-inconics. The poles of *L* wrt these conics are collinear on *L.**

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