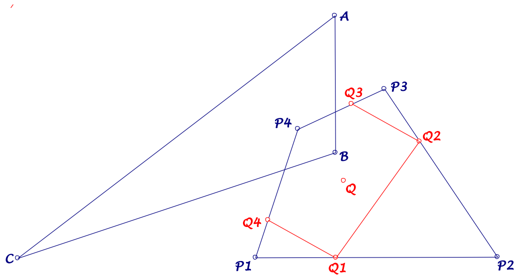
EQF-Note 2013-10-28

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Inscribed Parallelograms of a Quadrigon

For any point P in the plane of a quadrigon there is an inscribed parallelogram with center P. The centers of inscribed rhombi and inscribed rectangles lie on special conics. Their intersections give the centers of inscribed squares (see QG-L6 in EQF). This paper is a summary of
http://eckartschmidt.de/Pgive.pdf
Reference triangle for barycentric coordinates is QA-DT.



Inscribed Parallelograms

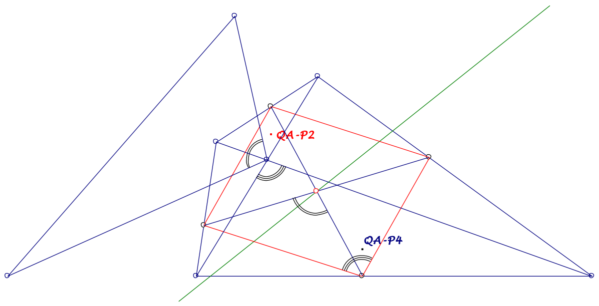
Let $P_1P_2P_3P_4$ be a quadrigon with diagonal triangle ABC as reference triangle for barycentric coordinates:

$$P_1(-p,q,r),\quad P_2(p,-q,r),\quad P_3(p,q,-r),\quad P_4(p,q,r)\,.$$

For a parallelogram center $Q(u:v:w)$ the inscribed parallelogram $Q_1Q_2Q_3Q_4$ can be constructed and calculated as follows: The intersection of P_3P_4 with the reflection of P_1P_2 in Q gives Q_3, and so on:

$$Q_1(p:-q:\frac{q^2u+p^2v+pqw}{qu-pv}),\quad Q_2(\frac{r^2v+q^2w+qru}{qw-rv}:-q:r)$$

$$Q_3(p:q:\frac{-q^2u-p^2v+pqw}{qu+pv}),\quad Q_4(\frac{-r^2v-q^2w+qru}{qw+rv}:q:r)\,.$$



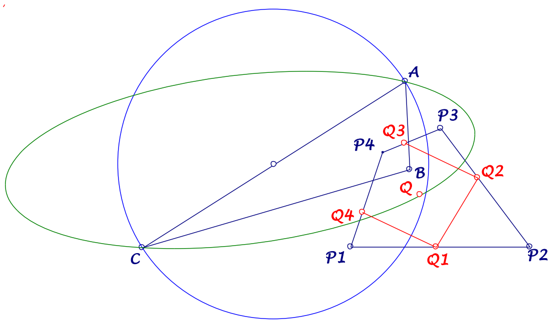
A special example is mentioned in *EQF*: The pedal quadrigon of the Isogonal Center *QA-P4* is a parallelogram. This parallelogram is a representative of parallelograms with center on the perpendicular bisector of *QA-P2.QA-P4*. These parallelograms have angles as in the intersection of the diagonals of the quadrigon.
The inscribed parallelogram degenerates for centers on the Nine-point Conic *QA-Co1*.

Inscribed Rhombi

A rhombus is a parallelogram with equal side lengths. The centers of inscribed rhombi lie on a conic with equation

$$q^2(p^2 S_A yz - q^2 S_B zx + r^2 S_C uv) + r^2 p^2 b^2 y^2 = 0\,.$$

This conic is the Involutary Conjugate *QA-Tf2* of the Thales circle about the third diagonal *AC* of the quadrigon.

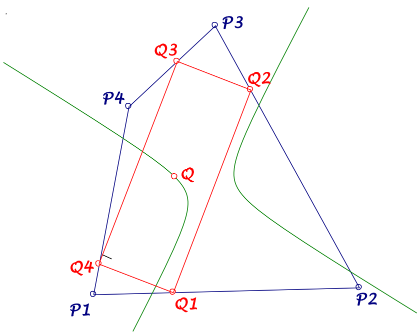


Special examples are two inscribed rhombi with sides parallel to the diagonals; the vertices divide the sides of the quadrigon in the ratio of the diagonal lengths.

Inscribed Rectangles

For rectangles the right angularity gives the equation for the centers:

$$a^2q^4r^2x^2 + b^2p^2r^2(p^2 - r^2)y^2 - c^2p^2q^4z^2 + 2p^2q^2r^2y(S_C x - S_A z) = 0$$



This conic contains the midpoints of the diagonals, for which the rectangles degenerate collinear. The center of the conic is the midpoint of the Euler-Poncelet Point *QA-P2* and the Isogonal Center *QA-P4*.

Inscribed Squares

There are two real intersections of the conics for the centers of inscribed rhombi and rectangles:

$$Q^{\pm}(p(S_A p \pm Sr : S_B q^2 : r(S_C r \pm Sp)).$$

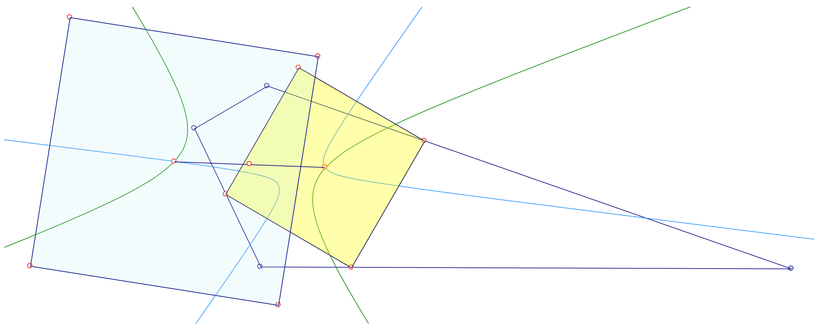
These centers of inscribed squares lie symmetric to the center of the conic for the centers of inscribed rhombi. Their connection is the Inscribed Square Axis *QG-L6* (see *EQF*). The vertices of the inscribed squares are:

$$Q_1^{\pm}(p : -q : \frac{S_C r^2 + c^2 pq \pm Sr(p+q)}{S_A p - S_B q \pm Sr}),$$

$$Q_2^{\pm}(\frac{S_A p^2 + a^2 qr \pm Sp(q+r)}{S_C r - S_B q \pm Sp} : -q : r),$$

$$Q_3^{\pm}(p : q : \frac{S_C r^2 - c^2 pq \pm Sr(p-q)}{S_A p + S_B q \pm Sr}),$$

$$Q_4^{\pm}(\frac{S_A p^2 - a^2 qr \pm Sp(r-q)}{S_C r + S_B q \pm Sp} : q : r).$$



For further properties have a look on my homepage 11.2.

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