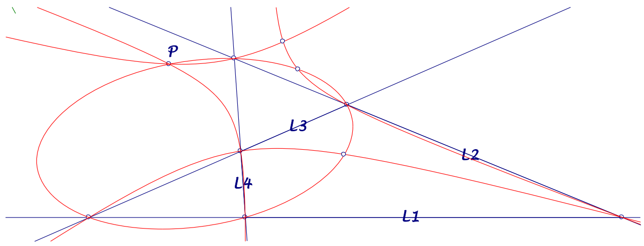
EQF-Note 2013-11-04

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Three *QG*-Circumconics for a Quadrilateral

For the three QG-components of a quadrilateral there are circumconics through a fixed point with three collinear fourth intersections. The corresponding line will be discussed here, leading to a new type of QL-cubics. − Reference triangle for barycentric coordinates is QL-DT.



The three *QG*-Circumconics and their Intersections

For a quadrilateral with lines

$$L_1(-l,m,n),\quad L_2(l,-m,n),\quad L_3(l,m,-n),\quad L_4(l,m,n)$$

there are the intersections

$$Q_{12}(m:l:0),\quad Q_{23}(0:n:m),\quad Q_{34}(-m:l:0),$$
$$Q_{41}(0:-n:m),\quad Q_{13}(n:0:l),\quad Q_{24}(-n:0:l)$$

and the quadrigon components

$$Q_{12}Q_{23}Q_{34}Q_{41},\quad Q_{12}Q_{24}Q_{34}Q_{13},\quad Q_{23}Q_{24}Q_{41}Q_{13}.$$

Let $P(u:v:w)$ be an arbitrary point, then the *QG*-circumconics through *P* have the equations

$$(l^2u^2-m^2v^2+n^2w^2)zx-(l^2x^2-m^2y^2+n^2z^2)wu=0,$$
$$(l^2u^2+m^2v^2-n^2w^2)xy-(l^2x^2+m^2y^2-n^2z^2)uv=0,$$
$$(-l^2u^2+m^2v^2+n^2w^2)yz-(-l^2x^2+m^2y^2+n^2z^2)vw=0.$$

The fourth intersections of two of these conics give

$$(m^2uv:l^2u^2-n^2w^2:-m^2vw),$$
$$(n^2uw:-n^2vw:l^2u^2-m^2v^2),$$
$$(n^2w^2-m^2v^2:-l^2uv:l^2wu).$$

These intersections are the Involutary Conjugates *QA-Tf2* of *P* wrt the *QG*-components. They are collinear on a line with the coefficients

$$L_P(l^2u(-l^2u^2+m^2v^2+n^2w^2),$$
$$m^2v(l^2u^2-m^2v^2+n^2w^2),n^2w(l^2u^2+m^2v^2-n^2w^2)).$$

Three Examples

P* = *QL-P1: The coefficients of $L_{QL\text{-}P1}$ are extensive. The line is the Clawson-Schmidt Conjugate *QL-Tf1* of the Dimidium Circle *QL-Ci6* containing *QL-P26*.

P* = *QL-P8: The line $L_{QL\text{-}P8}$ has the coefficients

$$L_{QL-P8}(l^2(-l^2+m^2+n^2), m^2(l^2-m^2+n^2), n^2(l^2+m^2-n^2))$$

and is the second asymptote of *QL-Co2* containing *QL-P23*.

P* = *QL-P13: The line $L_{QL\text{-}P13}$ has the coefficients

$$L_{QL-P13}(l^2m^2-m^2n^2+n^2l^2, l^2m^2+m^2n^2-n^2l^2, -l^2m^2+m^2n^2+n^2l^2)$$

and is a parallel to *QL-L9* through *QL-P19*.

For points *P* on L_i the line L_P is L_i again. For the vertices of the diagonal triangle *QL-DT* the line is the opposite sideline of *QL-DT*. For the midpoints of the sides of *QL-DT* the line containes the opposite vertex and *QL-P13*.

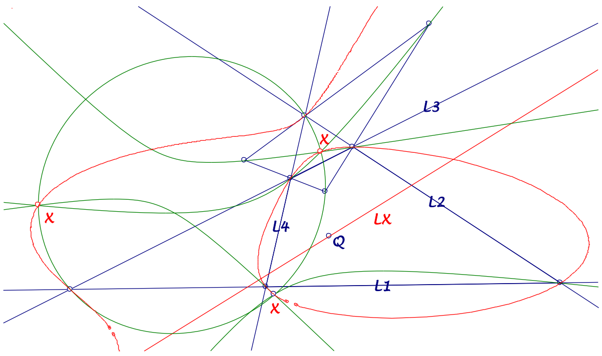
Further Properties

For a line *L* there are always three points *X* with $L_X = L$. These points are the common intersections of the conics, which are the *QA-Tf2* images of *L* wrt the *QG*-components.

For all lines through a fixed point $Q(x_o : y_o : z_o)$ these three points lie on a cubic with the equation

$$l^4 x_o x^3 + m^4 y_o y^3 + n^4 z_o z^3$$
$$-l^2m^2(x_o y + x y_o)xy - m^2n^2(y_o z + y z_o)yz - n^2l^2(z_o x + z x_o)zx = 0.$$

This cubic is the locus for all points *X*, whose L_X containes *Q*, passing through the six points of the quadrilateral.



If *Q* is a vertex of *QL-DT*, this conic is *QG-Co2* of the correspondent *QG*-component.

If *Q* = *QL-P13*, this cubic has the equation

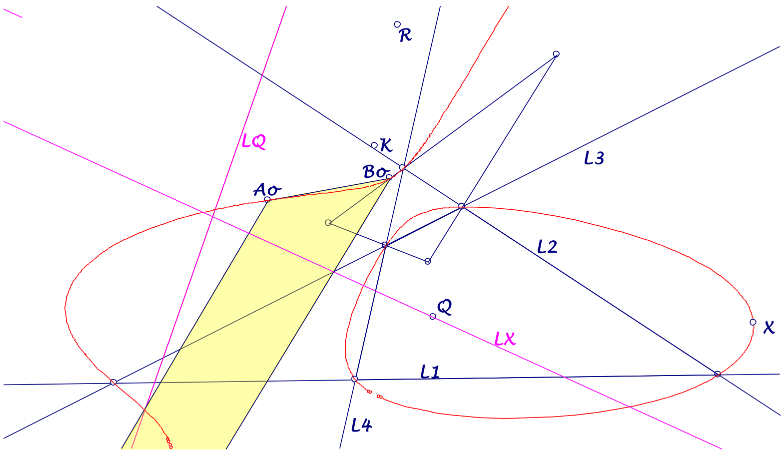
$$l^2(-x+y+z)x^2 + m^2(x-y+z)y^2 + n^2(x+y-z)z^2 = 0.$$

and is a circumcubic of the medial triangle of *QL-DT* , isotomic invariant wrt this triangle.

A new Type of *QL*-Cubics

The general case of the locus for points X, whose L_X containes a fixed point $Q(x_o : y_o : z_o)$

- … is a nonpivotal isocubic of type nK containing the points of the quadrilateral.
- The reference triangle $A_oB_oC_o$ for this cubic has its vertices in the third intersections of the cubic with the sidelines of *QL-DT*:
  $A_o(0:n^2z_o:m^2y_o)$, $B_o(n^2z_o:0:l^2x_o)$, $C_o(m^2y_o:l^2x_o:0)$.
  $A_oB_oC_o$ is the cevian triangle wrt *QL-DT* of a point
  $$K(m^2n^2y_oz_o:l^2n^2z_ox_o:l^2m^2x_oy_o),$$
  which is the image of Q wrt a *QL-DT*-isoconjugation with fixed points in the trilinear poles of L_i.
- The isoconjugation wrt $A_oB_oC_o$ for the cubic has fixed point K.
- The root R of the cubic is the trilinear pole of L_Q wrt $A_oB_oC_o$. The coordinates are extensive.



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