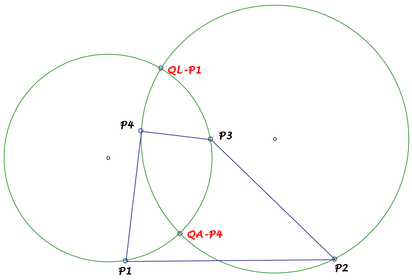
EQF-Note 2013-12-05

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Quartic for Quadrigons

The fixed points of a simple quadrigon transformation give an interesting quartic, anallagmatic and invariant wrt the Clawson-Schmidt Conjugate QL-Tf1. Construction and equation shall be given.



The transformation

Consider for a quadrigon circles through opposite vertices and a given point *P*. The second intersection of these circles shall be the image of *P*.

Let $P_1P_2P_3P_4$ be the quadrigon, $P_1P_2P_3$ the reference triangle for barycentric coordinates and $P_4(p:q:r)$. For a point $P(x:y:z)$ the image has somewhat extensive coordinates:

$$\left\{\frac{b^2\,(p+q+r)\,z\,(r\,x-p\,z)+c^2\,(p+r)\,X\,y+(a^2\,p+a^2\,r-b^2\,p)\,Z\,y}{a^2\,r\,Y\,z+b^2\,r\,X\,z+c^2\,r\,(p+q+r)\,x\,y-c^2\,p\,q\,z\,(x+y+z)},\right.$$

$$\frac{b^2\,y\,(x+y+z)}{c^2\,x\,y+b^2\,x\,z+a^2\,y\,z},$$

$$\left.\frac{b^2\,(p+q+r)\,x\,(p\,z-r\,x)+a^2\,(p+r)\,Z\,y+(-b^2\,r+c^2\,r+c^2\,p)\,X\,y}{b^2\,p\,Z\,x+c^2\,p\,Y\,x+a^2\,p\,(p+q+r)\,y\,z-a^2\,q\,r\,x\,(x+y+z)}\right\}$$

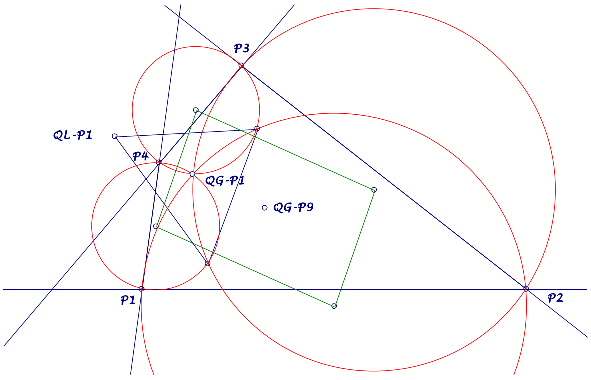
with

$$X:=x\,(q+r)-p\,(y+z);\;Y:=y\,(r+p)-q\,(z+x);\;Z:=z\,(p+q)-r\,(x+y)$$

Some properties:

- Fixed points of the transformation give a quartic (see below).
- The image of the Miquel Point *QL-P1* is the Isogonal Center *QA-P4* (see above).

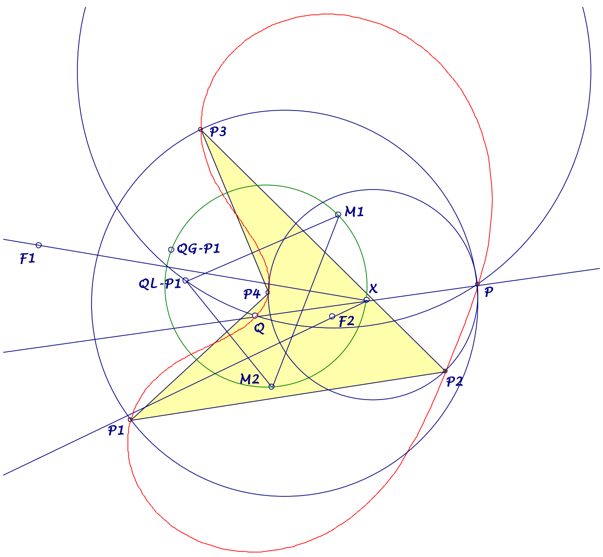
- The images of the intersections of opposite sidelines are vertices M_1 and M_2 (unequal *QL-P1*) of the Miquel Triangle *QA-Tr2*.
- The diagonals are invariant wrt this transformation.
- The images of the sidelines are circles, containing the vertices of the opposite side, the Diagonal Crosspoint *QG-P1* and a vertex (unequal *QL-P1*) of the Miquel Triangle.
- The midpoints of these circles give a parallelogram with center *QG-P9* and sidelines perpendicular to the diagonals of the quadrigon.



The Quartic of the Fixed Points

For fixed points *P* of the transformation the circles through opposite vertices of the quadrigon are tangent in *P*. These points give a quartic with the equation (*X*, *Y*, *Z* see above):

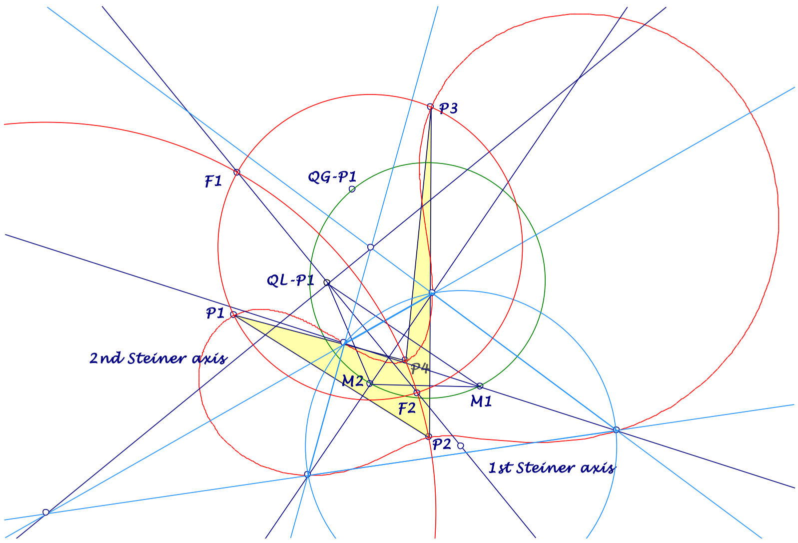
$$
\begin{aligned}
&(p+r)\, y^2 \,(a^4 z Z + c^4 x X) + a^2 c^2 (p+r)\, y^2 (x Z + z X) \\
&\quad - b^4 x z\, (r (x+y) X + p (y+z) Z) \\
&\quad - a^2 b^2 z\, (p Z y (z+y-x) + r (Z y (z-x) + Y (x+z)(y+x))) \\
&\quad - b^2 c^2 x\, (r X y (x+y-z) + p (X y (x-z) + Y (x+z)(y+z))) == 0
\end{aligned}
$$



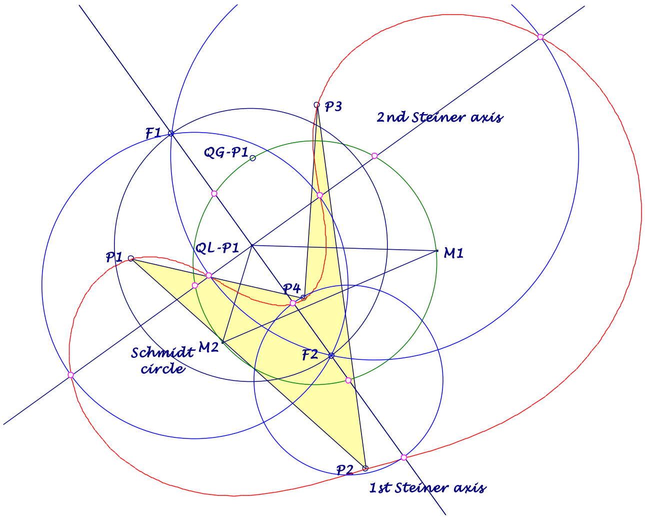
Construction of the quartic: For the construction we need the vertices M_1 and M_2 (unequal *QL-P1*) of the Miquel triangle and the fixed points F_1 and F_2 of the Clawson-Schmidt Conjugate *QL-Tf1*.

1. Circle through *QG-P1*, M_1, M_2 with a point X
2. L_x angle bisector of $\angle F_1XF_2$
3. P, Q as intersections of L_x and its image-circle wrt *QL-Tf1* are points of the quartic.

Properties:



- The quartic is the locus of points, where circles through opposite vertices of the quadrigon are tangent.
- The quartic is a circumquartic of the quadrigon.
- The quartic is invariant wrt *QL-Tf1*.
- For *QL-Tf1* partners on the quartic the midpoints lie on the circumcircle of M_1, M_2, *QG-P1*.
- The angle bisectors of the Miquel triangle at $M_{1,2}$ cut their *QL-Tf1* image on the quartic in points of a cyclic quadrigon.
- Midpoint of the circumcircle of this cyclic quadrigon is the excenter wrt *QL-P1* of the Miquel triangle.
- The quartic is anallagmatic, that means: The quartic is invariant wrt a reflection in a circle. There are two those circles with midpoints in the intersections of the opposite sidelines of the cyclic quadrigon (on the 2^{nd} Steiner axis), containing F_1 and F_2.
- The 1^{st} Steiner axis cuts the circle through M_1, M_2, *QG-P1* in two points. One point is the center of a circle orthogonal to the Schmidt circle, which cuts the 1^{st} Steiner line on the quartic.
- The 2^{nd} Steiner axis cuts the circle through M_1, M_2, *QG-P1* in two points. They are centers of circles through F_1 and F_2, which cut the 2^{nd} Steiner axis on the quartic.



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