EQF-Note 2014-01-12

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Alternating Sum of Distances

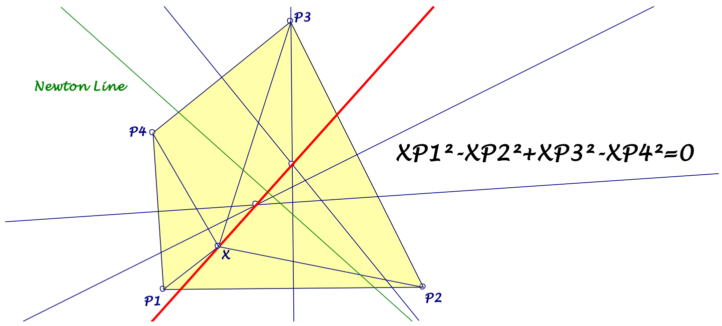
For quadrigons $P_1P_2P_3P_4$ with the sidelines L_1, L_2, L_3, L_4 here are discussed the loci for points X with the distance properties:

1. $XP_1^2 - XP_2^2 + XP_3^2 - XP_4^2 = 0$,
2. $XL_1^2 - XL_2^2 + XL_3^2 - XL_4^2 = 0$,
3. $XP_1 - XP_2 + XP_3 - XP_4 = 0$,
4. $XL_1 - XL_2 + XL_3 - XL_4 = 0$.

1. The locus for points X with the property

$$XP_1^2 - XP_2^2 + XP_3^2 - XP_4^2 = 0$$

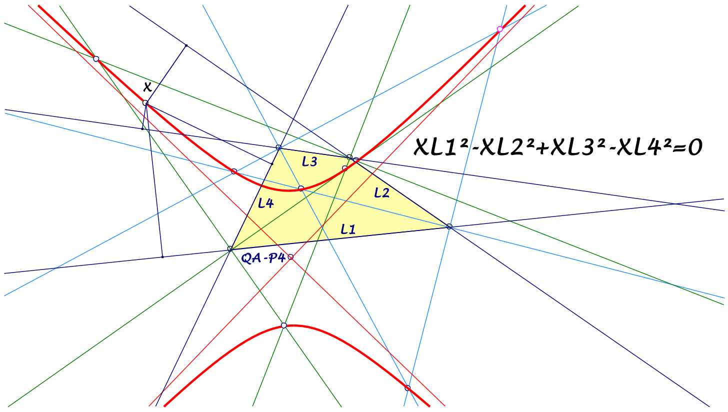
is a line, perpendicular to the Newton Line *QL-L1*, containing the intersections of the perpendicular bisectors of P_1P_2, P_3P_4 and P_2P_3, P_4P_1.
This line can easily be constructed and calculated.
Additional remark: For points X on a line, perpendicular to the Newton Line, the alternating sum is constant.



2. The locus for points X with the property

$$XL_1^2 - XL_2^2 + XL_3^2 - XL_4^2 = 0$$

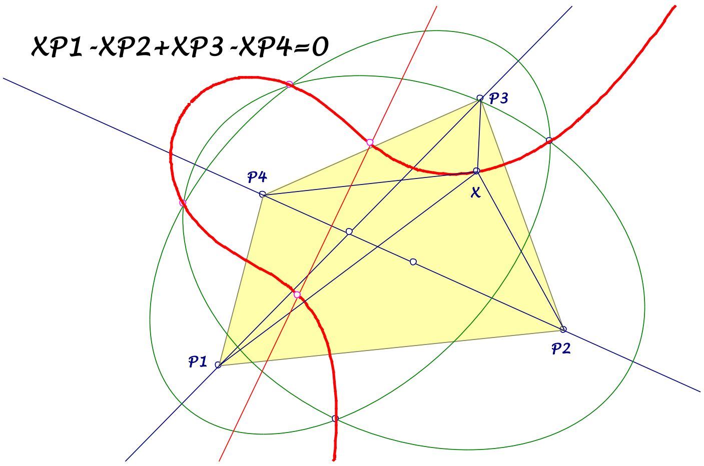
is a rectangular hyperbola centered in the Isogonal Center *QA-P4*. The hyperbola contains the intersections of the inner or outer angle bisectors at opposite vertices of the quadrigon (8 points, see 4.). So there is no problem to construct the curve and calculate its equation.



3. The locus for points X with the property

$$XP_1 - XP_2 + XP_3 - XP_4 = 0$$

can be constructed in the following way: Consider ellipses with foci P_1, P_3 and P_2, P_4 with the same length of the principal axis. The intersections give points of a curve, which seems to be a cubic, but a calculation gives a (very extensive) equation of degree 5 (Mathematica gives no further factorization).



The intersections of the perpendicular bisectors of P_1P_2, P_3P_4 and P_2P_3, P_4P_1 (see 1.) lie on the curve. The corresponding line is parallel to the asymptote of the curve.

4. The loci for points X with the property

$$XL_1 - XL_2 + XL_3 - XL_4 = 0$$

are sections of lines, connecting intersections of angle bisectors of opposite vertices (see 2.).

If we denote the angle bisectors at P_i with g_i^+ and g_i^-, we get the intersections

$$P_{++}, P_{+-}, P_{-+}, P_{++} \quad e.g. \quad P_{+-} = g_1^+ \cap g_3^-,$$

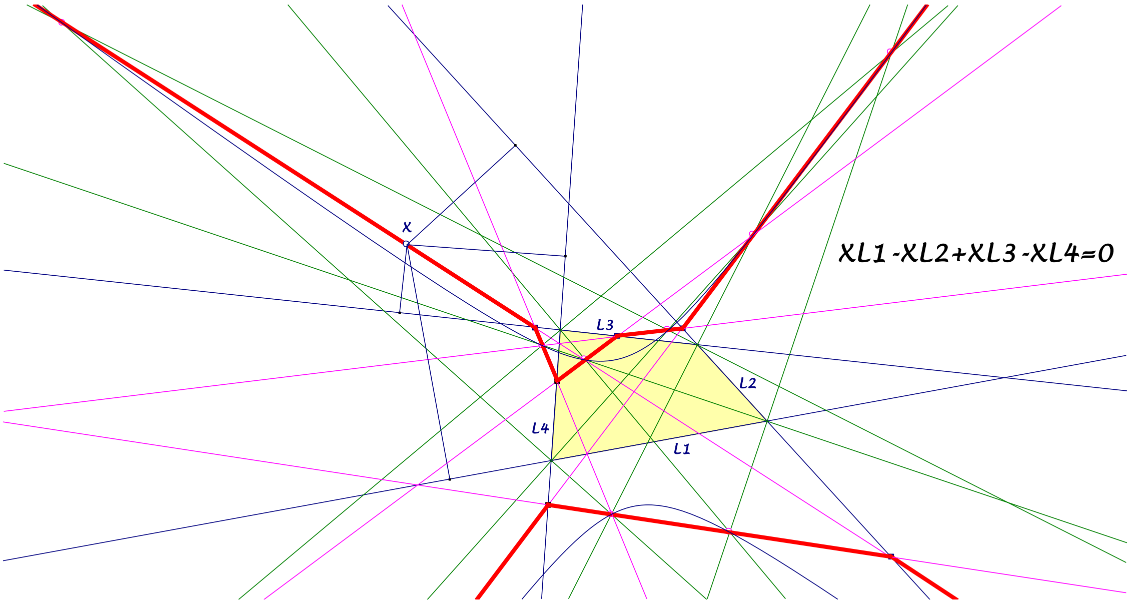
$$Q_{++}, Q_{+-}, Q_{-+}, Q_{++} \quad e.g. \quad Q_{+-} = g_2^+ \cap g_4^-.$$

These are the points on the rectangular hyperbola of 2, also satisfying the condition 4.

On each of the six lines

$$P_{++}Q_{--},\ P_{+-}Q_{+-},\ P_{+-}Q_{-+},\ P_{-+}Q_{+-},\ P_{-+}Q_{-+},\ P_{--}Q_{++}$$

we find one section for the points X, bordered by intersections with sidelines of the quadrigon. In the other sections another sign constellation in the sum gives zero.



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