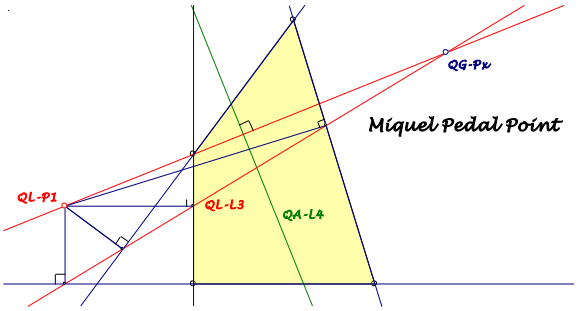
EQF-Note 2014-01-20

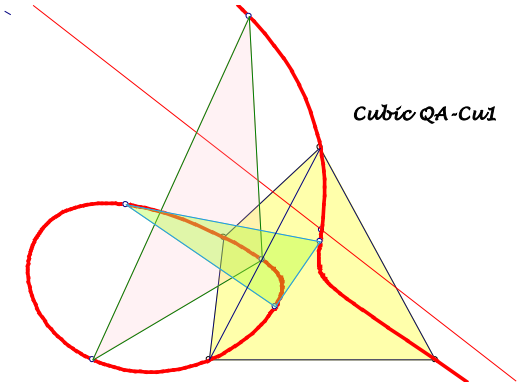
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

The Miquel Pedal Point

For the Miquel Point of a quadrilateral the pedal points on the sidelines are collinear on the Pedal Line QL-L3. For a quadrigon a special point on this line will be discussed: the intersection of QL-L3 and the perpendicular line through the Miquel Point wrt QA-L4.

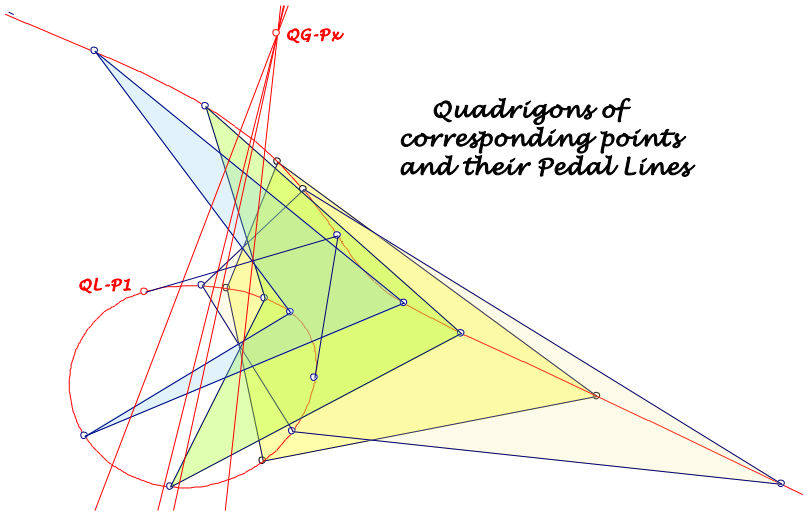


Background for this point is the cubic *QA-Cu1* for quadrangles, a pivotal isogonal cubic wrt the Miquel Triangle *QA-Tr1*. The asymptote is parallel *QA-L4*. The cubic contains the vertices of the quadrangle, the vertices of the *QA*-Diagonal Triangle *QA-Tr1* and the vertices of the Miquel Triangle *QA-Tr2*.



This cubic is invariant under the three versions of the Clawson-Schmidt Conjugate *QL-Tf1*. Any point on the cubic has three images wrt *QL-Tf1*, forming a "quadrangle of corresponding points" on the cubic with a common tangential point on the cubic and the same Miquel Triangle as the reference quadrangle. Examples: Evidently the vertices of the quadrangle, further the in- and excenters of the Miquel Triangle, the vertices of the *QA*-Diagonal Triangle and the Isogonal Center *QA-P4*, the vertices of the Miquel Triangle *QA-Tr2* and the point at infinity of the asymptote.

Now we consider a quadrigon: Taken as quadrilateral, we get the Miquel Point *QL-P1*. Taken as quadrangle, we get the cubic *QA-Cu1* with quadrangles of corresponding points. Each quadrangle of corresponding points has a quadrigon component with the same Miquel Point as the reference quadrigon, only these quadrigons of corresponding points shall be considered:



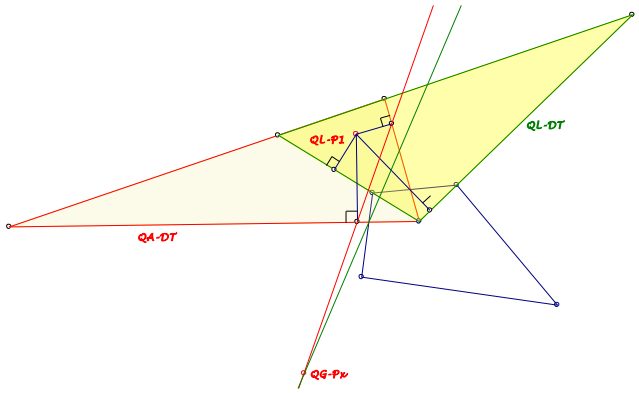
The Pedal Lines (*QL-L3*) for quadrigons of corresponding points on the cubic *QA-Cu1* of a reference quadrigon have a common point.

If we take the *QL*-Diagonal Triangle *QL-Tr1* as reference triangle for barycentric coordinates with $L_4 = (l, m, n)$, this point is

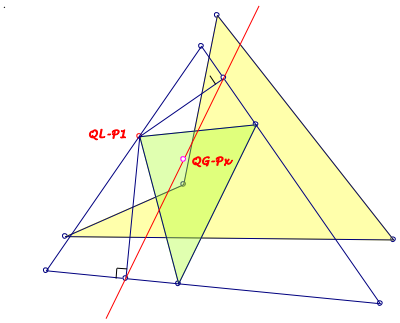
$$(M^2S_B{}^2 + N^2S_C{}^2 - L(N-L)S_AS_B + 2N^2S_BS_C - NLS_AS_C$$
$$: -(LS_A - NS_C)^2 - MS_B(LS_A - 2MS_B + NS_C)$$
$$: L^2S_A{}^2 + M^2S_B{}^2 - N(L-N)S_BS_C + 2L^2S_AS_B - NLS_AS_C)$$
$$\text{with} \quad L = m^2 - n^2, \quad M = n^2 - l^2, \quad N = l^2 - m^2 .$$

Properties

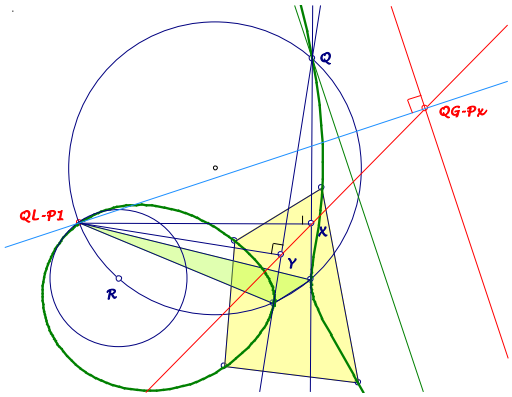
- *QG-Px* lies on the Pedal Line *QL-L3* of the quadrigon.
- *QG-Px* lies on a perpendicular line through the Miquel Point wrt *QA-L4*.
- *QG-Px* is a point on the line through the pedal points of the Miquel Point wrt the legs of the *QA*-Diagonal Triangle.
- *QG-Px* is a point on the perpendicular bisector of the pedal points of the Miquel Point wrt the legs of the *QL*-Diagonal Triangle.



- The Miquel Point is a point on a sideline of the excenter triangle of the Miquel Triangle. The line through the pedal points of the Miquel Point wrt the other two sidelines contains *QG-Px*.



- Let *Q* be the intersection of *QA-Cu1* and its asymptote and *X*, *Y* the pedal points of the Miquel Point to the connections of *Q* with the other two vertices of the Miquel Triangle, then *QG-Px* is a point of *XY*.
- Let *R* be the reflection of *Q* in the circumcenter of the Miquel Triangle and *Ci* a circle round *R* through the Miquel Point, then the Clawson-Schmidt Conjugate *QL-Tf1* of *Ci* is a line through *QG-Px* (parallel to the asymptote) and *QG-Px* is the pedal point of *QL-P1* wrt this line.



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