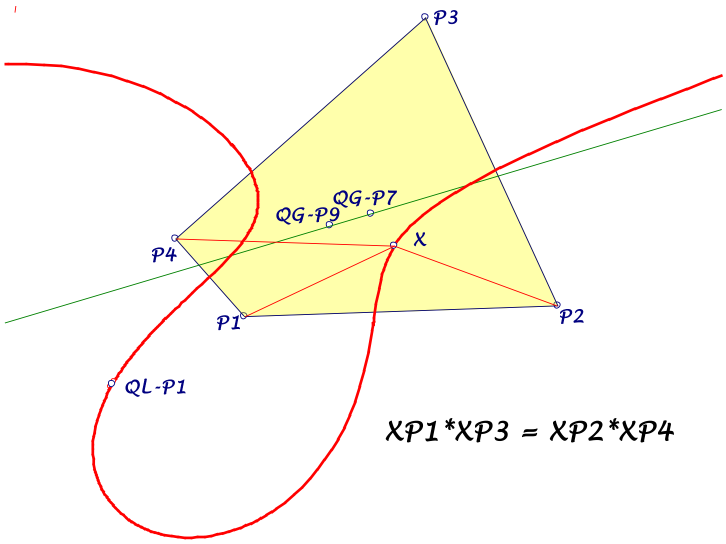
EQF-Note 2014-01-23

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

QG-Cubic with $XP_1 \cdot XP_3 = XP_2 \cdot XP_4$

The Miquel Point has the property, that the products of distances to opposite vertices of a quadrigon are equal. In general: Points with this property lie on a cubic, invariant under the Clawson-Schmidt Conjugate QL-Tf1. A construction of the cubic is possible related to the line QG-P7.QG-P9.



Points X in the QG-environment with the property

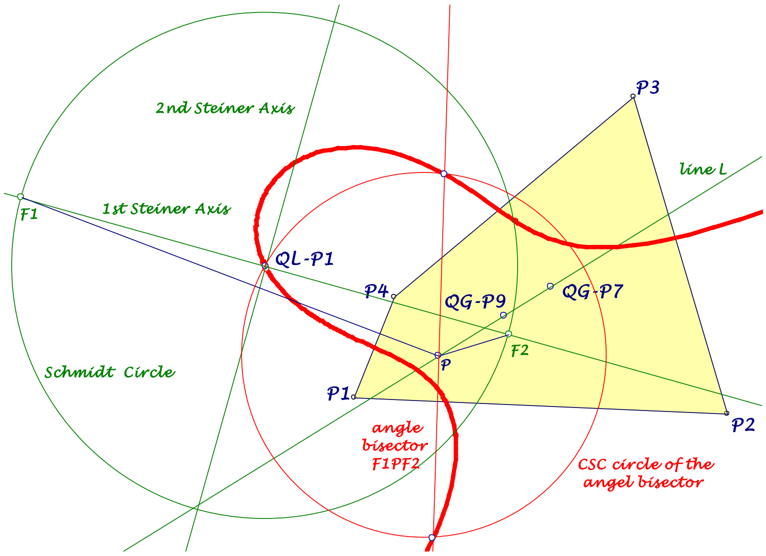
$$XP_1 \cdot XP_3 = XP_2 \cdot XP_4$$

lie on a cubic. Using the QL-Diagonal Triangle QL-$Tr1$ as reference triangle for barycentric coordinates, the equation of the cubic can easily be calculatet, but it is rather extensive and here not printed. There is one point at infinity

$$(-a^2l^2+S_Cm^2+S_Bn^2 : S_Cl^2-b^2m^2+S_An^2 : S_Bl^2+S_Am^2-c^2n^2),$$

which is the point at infinity of the line L = *QG-P7.QG-P9*.

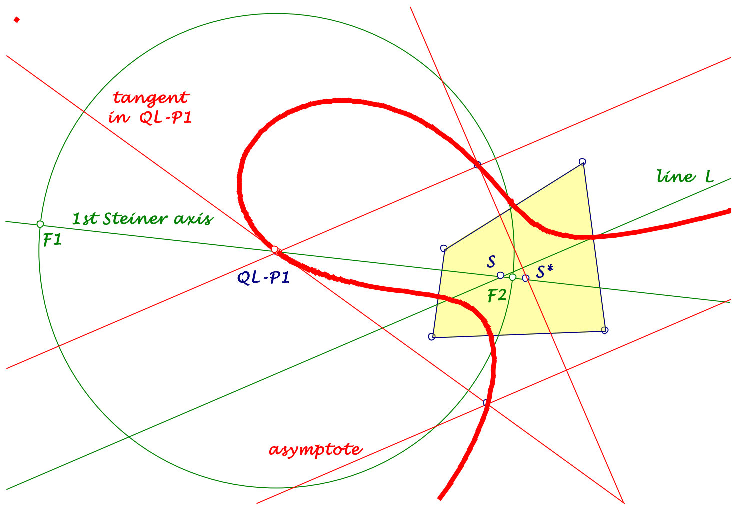
For a construction of the cubic we need (see *EQF*, *QL-Tf1*) the Clawson-Schmidt Conjugate (*CSC*) and its fixed points F_1 and F_2, the Schmidt Circle, the 1st and 2nd Steiner Axis and the Miquel Point *QL-P1*. For variable points P on the line L = *QG-P7.QG-P9* the intersections of the angle bisector of $\angle F_1PF_2$ and its *CSC*-image (a circle through *QL-P1*) are points of the cubic.



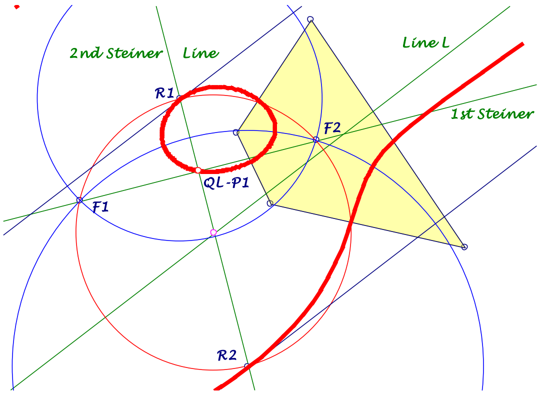
Properties

Let *S* be the intersection of the line *L* = *QG-P7.QG-P9* and the 1st Steiner Axis and *S** its reflection in the Schmidt Circle.

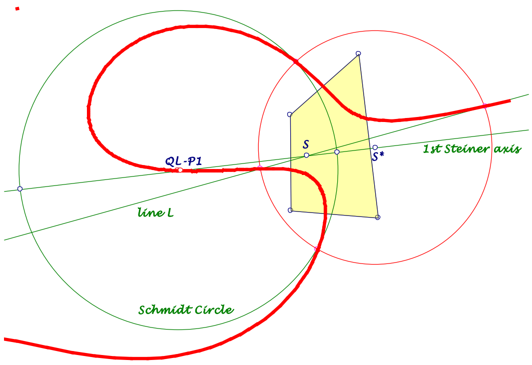
1. The cubic contains the Miquel Point *QL-P1*.
2. The cubic is invariant wrt the Clawson-Schmidt Conjugate *QL-Tf1*.
3. The cubic is unipartite, if *S* lies inside F_1F_2; the cubic is bipartite, if *S* lies outside F_1F_2.
4. The asymptote is a parallel to the line *L* through the reflection of the Miquel Point in the line *L*.
5. A parallel to the line *L* through the Miquel point, reflected in the 1st Steiner Axis gives the tangent to the cubic in the Miquel Point.
6. The tangent in the Miquel Point and the asymptote intersect on the cubic.
7. A parallel to the line *L* through the Miquel Point and a perpendicular to *L* through *S** intersect on the cubic.



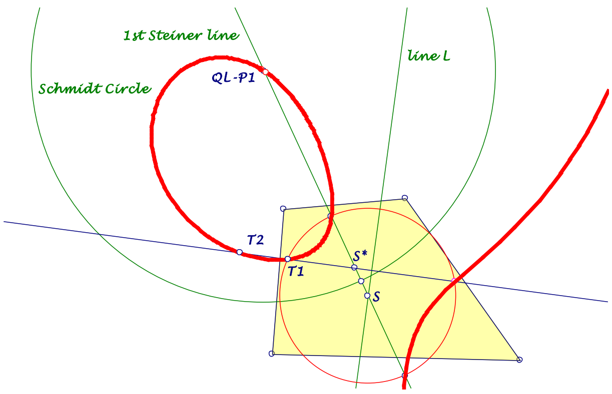
8. The intersections R_1 and R_2 of the cubic and the 2nd Steiner Line lie on a circle through F_1 and F_2 round the intersection of the line L and the 2nd Steiner Line.
9. The cubic is anallagmatic, invariant under reflections in circles round R_1 or R_2 through F_1 and F_2.



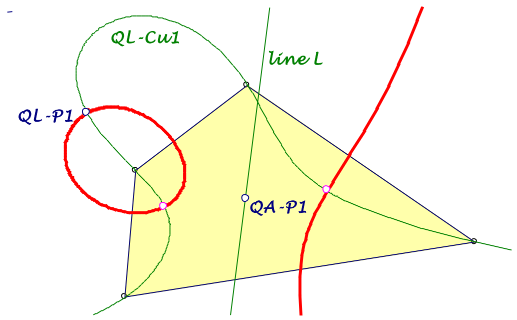
10. If the cubic is unipartite, there is a circle perpendicular to the Schmidt Circle and centered in S^*, containing the intersections of the cubic and the Schmidt Circle and the intersections of the cubic and the line L.



11. If the cubic is bipartite, there is a circle perpendicular to the Schmidt Circle and centered in S, containing the intersections of the cubic and the 1st Steiner Axis (without $QL\text{-}P1$) and the intersections T_1 and T_2 of the cubic and the perpendicular to L through S^* (without the point out of 7).



12. There are alternative constructions of the cubic: If S is inside F_1F_2, circles round variable points X on L and perpendicular to the circle out of 10 cut the connections of X and the Miquel Point in points of the cubic. If S is outside F_1F_2, circles round variable points X on L through T_1 and T_2 cut the connections of X and the Miquel Point in points of the cubic.
13. The cubic intersects the *QL*-Quasi Isogonal Cubic *QL-Cu1* perpendicular in three points, the Miquel Point and two *CSC*-partners symmetric wrt *QA-P1*.



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