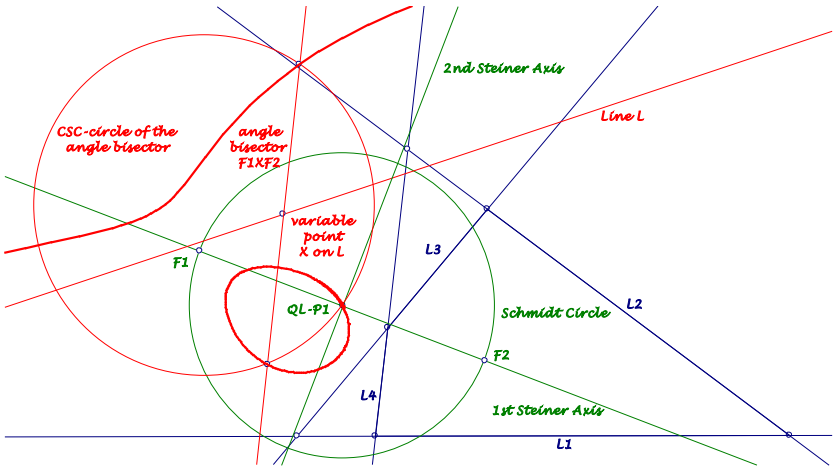
EQF-Note 2014-01-31

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Cubics related to Lines

For lines a special construction in the QL-environment gives a cubic through the Miquel Point QL-P1, invariant wrt the Clawson-Schmidt Conjugate QL-Tf1. For example: The Newton Line QL-L1 gives the QL-Quasi Isogonal Cubic QL-Cu1. – The relationships are only CABRI-controlled.



The Construction

We consider a reference quadrilateral with the lines L_1, L_2, L_3, L_4 and a further line L. For the construction of a cubic for the line L we need (see *EQF*, *QL-Tf1*) the Clawson-Schmidt Conjugate (*CSC*) and its fixed points F_1 and F_2, the Schmidt Circle, the 1st and 2nd Steiner Axis and the Miquel Point *QL-P1* of the quadrilateral.

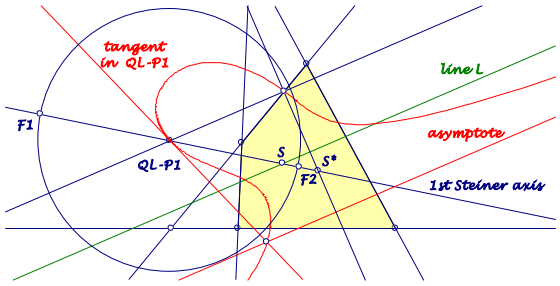
For variable points *X* on the line *L* the intersections of the angle bisector of $\angle F_1XF_2$ and its *CSC*-image (a circle through *F*) are points of the cubic.

Properties

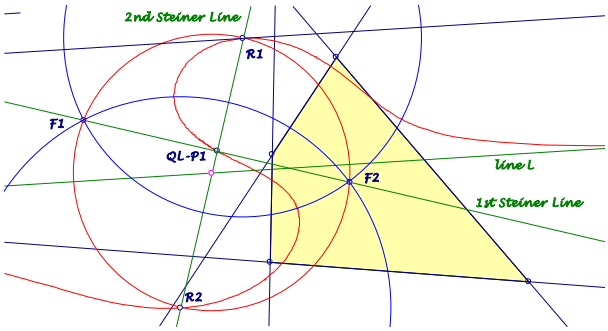
Let *S* be the intersection of the line *L* and the 1st Steiner Axis and *S** its reflection in the Schmidt Circle.

1. The cubic contains the Miquel Point *QL-P1*.
2. The cubic is invariant wrt the Clawson-Schmidt Conjugate *QL-Tf1*.

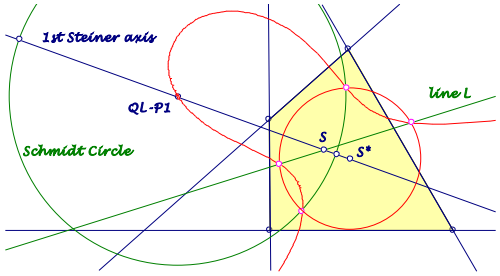
3. The cubic is unipartite, if S lies inside F_1F_2; the cubic is bipartite, if S lies outside F_1F_2.
4. The asymptote is a parallel to the line L through the reflection of the Miquel Point in the line L.
5. A parallel to the line L through the Miquel point, reflected in the 1st Steiner Axis gives the tangent to the cubic in the Miquel Point.



6. The tangent in the Miquel Point and the asymptote intersect on the cubic.
7. A parallel to the line L through the Miquel Point and a perpendicular to L through S^* intersect on the cubic.
8. The intersections R_1 and R_2 of the cubic and the 2nd Steiner Line lie on a circle through F_1 and F_2 round the intersection of the line L and the 2nd Steiner Line.

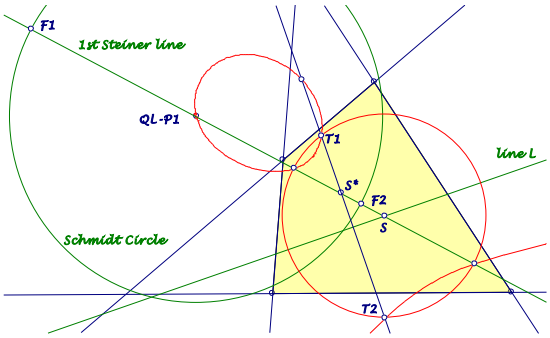


9. The cubic is anallagmatic, invariant under reflections in circles round R_1 or R_2 through F_1 and F_2.
10. If the cubic is unipartite, there is a circle perpendicular to the Schmidt Circle and centered in S^*, containing the intersections of the cubic and the Schmidt Circle and the intersections of the cubic and the line L.

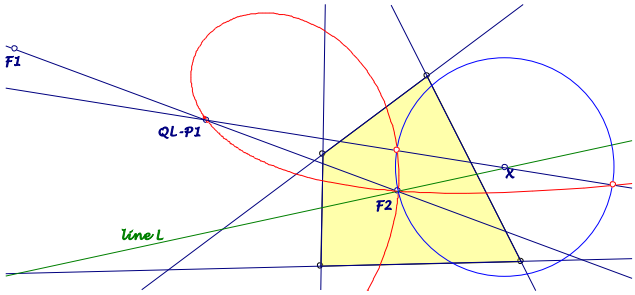


11. If the cubic is bipartite, there is a circle perpendicular to the Schmidt Circle and centered in S, containing the

intersections of the cubic and the 1st Steiner Axis (without *QL-P1*) and the intersections T_1 and T_2 of the cubic and the perpendicular to *L* through *S** (without the point out of 7).



12. There are alternative constructions of the cubic: If *S* is inside F_1F_2, circles round variable points *X* on *L* and perpendicular to the circle out of 10 cut the connections of *X* and the Miquel Point in points of the cubic. If *S* is outside F_1F_2, circles round variable points *X* on *L* through T_1 and T_2 out of 11 cut the connections of *X* and the Miquel Point in points of the cubic.
13. For the 1st Steiner Axis the cubic degenerates to the Schmidt Circle and the 1st Steiner Line.
14. **For the Newton Line *QL-L1* the cubic is the *QL*-Quasi Isogonal Cubic *QL-Cu1*.**
15. For lines *L* through one of the fixed points F_i, the cubic is a strophoid of the line *L* with the pole *QL-P1* and the fixed point F_i. That means, that the intersections of circles through F_i round variable points *X* on the line *L* and the connections of *X* with the Miquel Point generate the cubic.



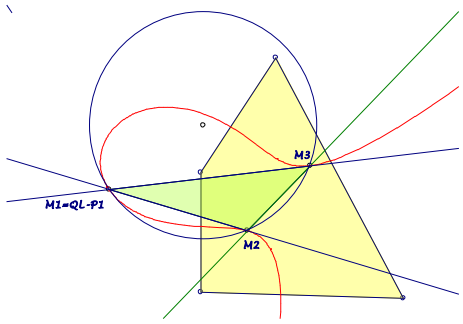
16. For lines *L* through the Miquel Point the cubic is symmetric wrt *QL-P1*, cutting the 2nd Steiner Line on the Schmidt Circle.

The Cubic for Quadrigons

Here are some remarks for quadrigons, considering the Miquel Triangle $M_1M_2M_3$ (*QA-Tr2*) with Miquel Point M_1.

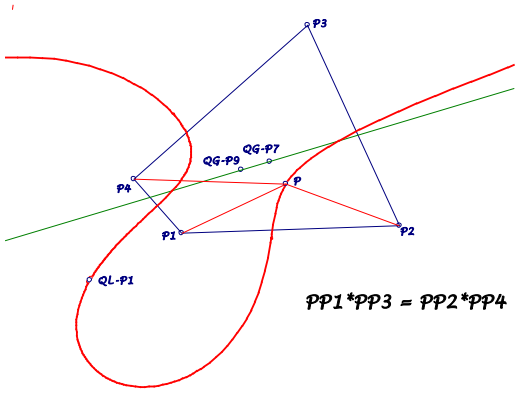
Cubic related to M_2M_3

- For the sideline M_2M_3 of the Miquel Triangle we get a circumcubic of the Miquel Triangle.
- This cubic is the locus for points, whose Clawson-Schmidt Conjugate *QL-Tf1* is the isogonal conjugate wrt the Miquel Triangle.



- M_1M_2 and M_1M_3 are tangent to the cubic.
- For the Miquel Point the tangent to the cubic (see 5) is also the tangent to the circumcircle of the Miquel Triangle.

Cubic related to *QG-P7.QG-P9*



- This cubic is the locus for points, whose distance products wrt opposite vertices are equal (see message 414).

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