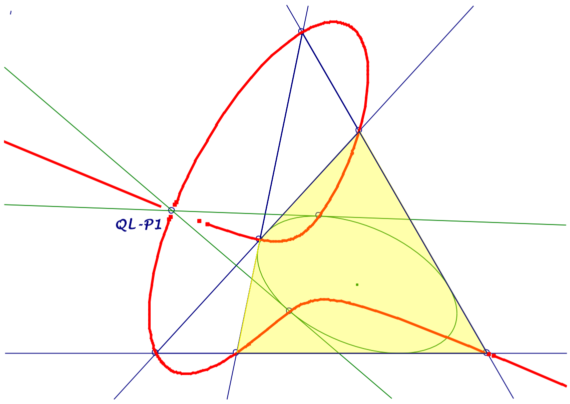
EQF-Note 2014-02-27

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Nodal *QL*-Cubic wrt the Miquel Point

If we consider a quadrilateral and the tangents from the Miquel Point to inscribed conics, the points of tangency will give a cubic, containing the six points of the quadrilateral and the Miquel Point as nodal point. The cubic is a conico-pivotal isocubic.



Let the lines L_1, L_2, L_3, L_4 define a quadrilateral with a reference triangle L_1, L_2, L_3 for barycentric coordinates and a fourth line $L_4(l,m,n)$.
We consider inscribed conics of the quadrilateral, centered on the Newton Line, and the tangents from the Miquel Point *QL-P1* to these conics. The points of tangency give a cubic, which shall be discussed here.

Properties:

- The equation of the cubic:

$$2\,l\,m\,n\,L\,M\,N\,(a^2 b^2 N + a^2 c^2 M + b^2 c^2 L)\,x\,y\,z$$
$$+\,a^4\,m\,n\,M^2 N^2\,(m\,y + n\,z)\,y\,z$$
$$+\,b^4\,n\,l\,N^2 L^2\,(n\,z + l\,x)\,z\,x$$
$$+\,c^4\,l\,m\,L^2 M^2\,(l\,x + m\,y)\,x\,y == 0$$

with $L = m - n,\ M = n - l,\ N = l - m.$

- The cubic contains the six intersections of the four lines of the quadrilateral.
- The cubic contains the Miquel Point *QL-P1*.
- The Miquel Point *QL-P1* is a nodal point with orthogonal tangents.
- The tangents at the Miquel Point are the Steiner Axes (see *QL-Tf1*).

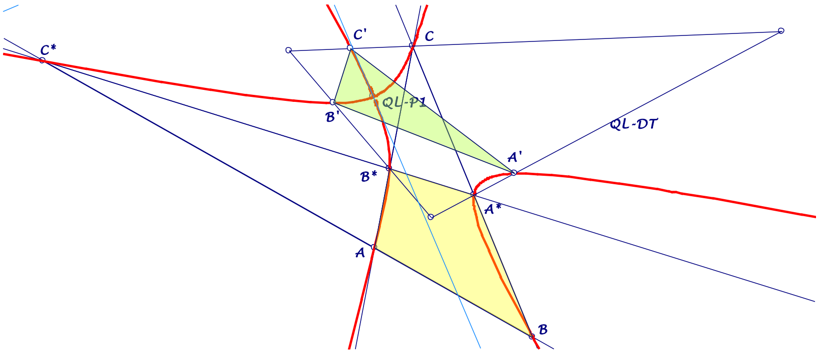
- The cubic contains the vertices of the Ceva triangle *Tr* = *A´B´C´* of *QL-P1* wrt the *QL*-Diagonal Triangle *QL-DT*.

$$A'\left(\frac{2a^2MN+b^2LN+c^2LM}{(c^2M-b^2N)\,l\,L},\ -\frac{1}{m},\ \frac{1}{n}\right)$$

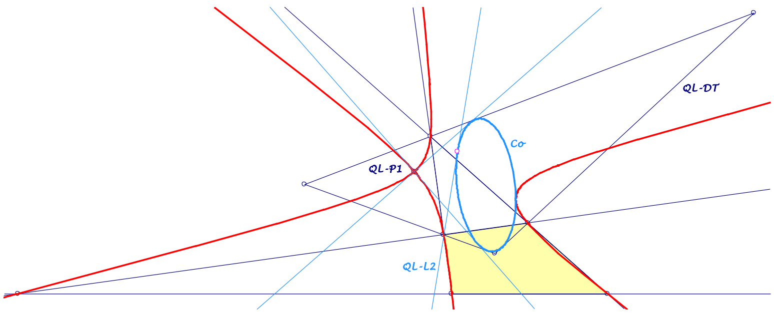
$$B'\left(\frac{1}{l},\ \frac{a^2MN+2b^2LN+c^2LM}{(a^2N-c^2L)\,m\,M},\ -\frac{1}{n}\right)$$

$$C'\left(-\frac{1}{l},\ \frac{1}{m},\ \frac{a^2MN+b^2LN+2c^2LM}{(b^2L-a^2M)\,n\,N}\right)$$

- The cubic is the locus for the intersections of lines through *QL-P1* and their *QL-Tf2*-image.



- The cubic is invariant under an isoconjugation * wrt the reference triangle *Tr* = *A´B´C´* and the fixed point *QL-P1*.
- The isoconjugation * swaps the points of tangency for tangents from *QL-P1* to inscribed conics.
- The isoconjugation * swaps opposite points of the quadrilateral.
- The polars of *QL-P1* wrt inscribed conics are tangents of an inscribed conic *Co* of the *QL*-Diagonal Triangle *QL-DT*.



- *Co* is also the envelope of the *QL-Tf2* images of lines through *QL-P1*.
- Equation of Co:

$$U^2x^2+V^2y^2+W^2z^2-2VWyz-2WUzx-2UVxy=0;$$

$$U:=l^2(m^2-n^2)(b^2(l^2-m^2)+c^2(n^2-l^2));$$

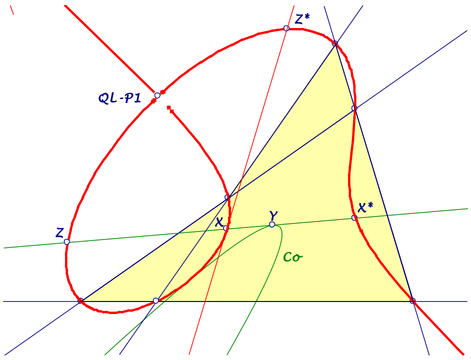
$$V:=m^2(n^2-l^2)(c^2(m^2-n^2)+a^2(l^2-m^2));$$

$$W:=n^2(l^2-m^2)(a^2(n^2-l^2)+b^2(m^2-n^2)).$$

- Special tangents of *Co* are the Steiner Axes (see above) and the Steiner Line *QL-L2*. Point of tangency wrt *QL-L2*:

$$\{l^4 (m^2 - n^2)^2 V W, m^4 (n^2 - l^2)^2 W U, n^4 (l^2 - m^2)^2 U V\}$$

- If the tangents from *QL-P1* to the cubic contact in *X* and *X** and *XX** is tangent in *Y* to the conic *Co*, than the fourth harmonic point *Z* is the third intersection of *XX** with the cubic.

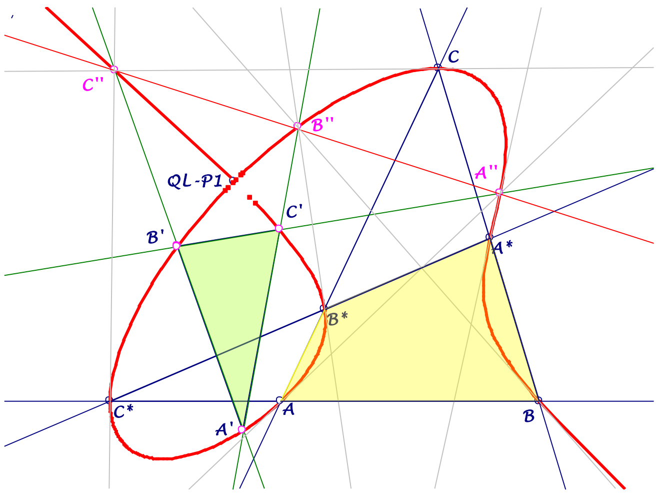


- Two isoconjugate points *X* and *X** on the cubic have the same tangential, which is the isoconjugate *Z** of the third intersection of *XX** with the cubic.
- The third intersections *A´´*, *B´´*, *C´´* of the sidelines of the triangle *Tr* = *A´B´C´* with the cubic are collinear:

$$A'' \left(\frac{a^4 m n M^2 N^2 (-c^2 M + b^2 N)}{2 a^2 N M + b^2 N L + c^2 M L}, b^4 n l N^2 L, -c^4 l m L M^2\right)$$

$$B'' \left(-a^4 m n M N^2, \frac{b^4 n l N^2 L^2 (-a^2 N + c^2 L)}{a^2 N M + 2 b^2 N L + c^2 M L}, c^4 l m L^2 M\right)$$

$$C'' \left(a^4 m n M^2 N, -b^4 n l N L^2, \frac{c^4 l m L^2 M^2 (-b^2 L + a^2 M)}{a^2 N M + b^2 N L + 2 c^2 M L}\right)$$



- The points *A´´*, *B´´*, *C´´* are the tangentials of opposite points of the quadrilateral.
- These properties show, that the cubic is a nonpivotal isocubic; more precisely: The cubic is a conico-pivotal isocubic *cK* with pivotal-conic *Co* (see *B. Gibert: Special Isocubics*, 1.5 and 8.1).
- Finally: The cubic has one inflection point; its isoconjugate gives a point of tangency between the cubic and its pivotal-conic *Co*.

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