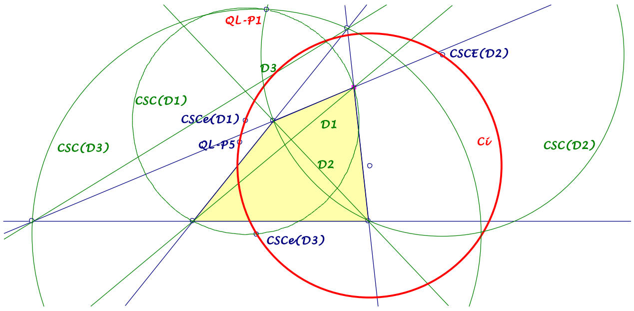
EQF-Note 2014-03-04

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

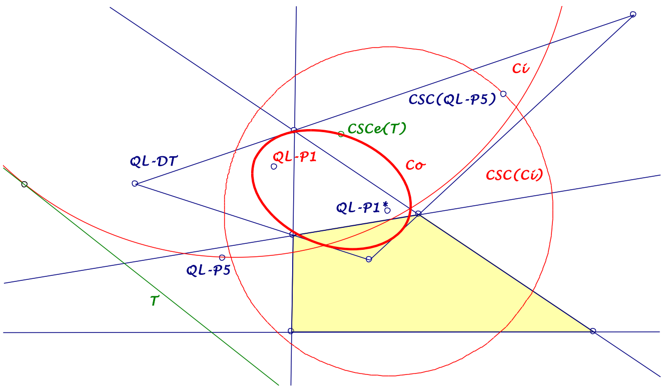
A Network of QL-Transformations

CSC is a shortcut for the transformation QL-Tf1 (Clawson-Schmidt Conjugate). CSCe is a transformation, giving for a line the CSC-image of the reflection of the Miquel Point QL-P1 in the line (see QFG messages 435, 436). Here are gathered some relationships to the QL-Line Conjugate QL-Tf2.

CSC is a point-to-point transformation, which swaps opposite points of a quadrilateral. Lines will give circles through the Miquel Point *QL-P1*. The four lines of a quadrilateral give circumcircles of the triangle components.
CSCe is a line-to-point transformation. For a line of the quadrilateral *CSCe* gives the circumcenter of the remaining triangle component (on the Miquel Circle *QL-Ci3*). A line pencil wrt a point gives a line, whose *CSCe*-image is the point again.
QL-Tf2 is a line-to-line transformation with the lines of the quadrilateral as fixed lines.

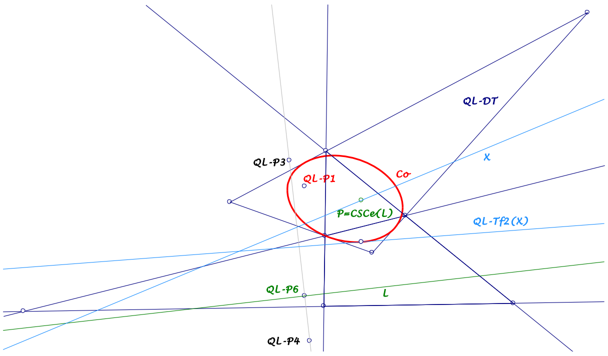


- The *CSCe*-images of the *QL*-diagonals lie on a circle *Ci* through the Clawson Center *QL-P5*. (These points are the circumcenters for the *CSC*-image-circles of the diagonals.)



- *CSC(Ci)* gives a circle through *CSC(QL-P5)* centered in the isogonal conjugate *QL-P1** of the Miquel Point wrt *QL-DT*.
- *CSC(QL-P1*)* is the reflection of *QL-P1* in *Ci*.
- The *CSCe*-images of tangents *T* at the circle *Ci* give an inscribed conic *Co* of *QL-DT* with foci *QL-P1* and *QL-P1**.
- The conic *Co* is also the envelope of the perpendicular bisectors of *QL-P1* and points of *CSC(Ci)*.
- The equation of *Co* in *DT*-coordinates:

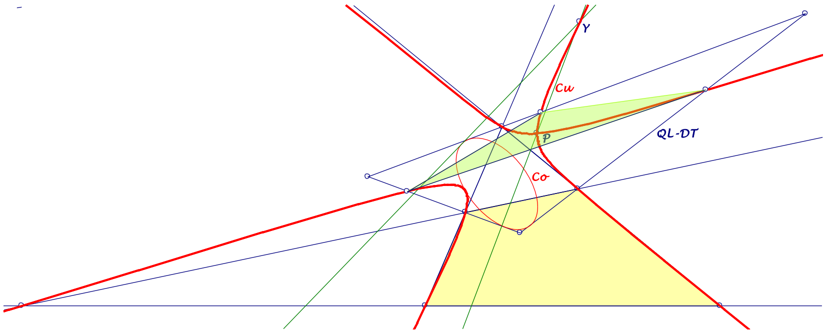
$$U^2 x^2 + V^2 y^2 + W^2 z^2 - 2\,U V x y - 2\,V W y z - 2\,W U z x = 0,$$
$$U = L\ (-L\,Sa + M\,Sb + N\,Sc)$$
$$(L^2\,Sa^2 + M^2\,Sb^2 + N^2\,Sc^2 + 2\ (M - N)\ Sa\ (M\,Sb - N\,Sc) - 2\,M\,N\,Sb\,Sc),$$
$$V = M\ (L\,Sa - M\,Sb + N\,Sc)$$
$$(L^2\,Sa^2 + M^2\,Sb^2 + N^2\,Sc^2 + 2\ (N - L)\ Sb\ (N\,Sc - L\,Sa) - 2\,N\,L\,Sc\,Sa),$$
$$W = N\ (L\,Sa + M\,Sb - N\,Sc)$$
$$(L^2\,Sa^2 + M^2\,Sb^2 + N^2\,Sc^2 + 2\ (L - M)\ Sc\ (L\,Sa - M\,Sb) - 2\,L\,M\,Sa\,Sb),$$
$$L = m^2 - n^2,\ M = n^2 - l^2,\ N = l^2 - m^2.$$



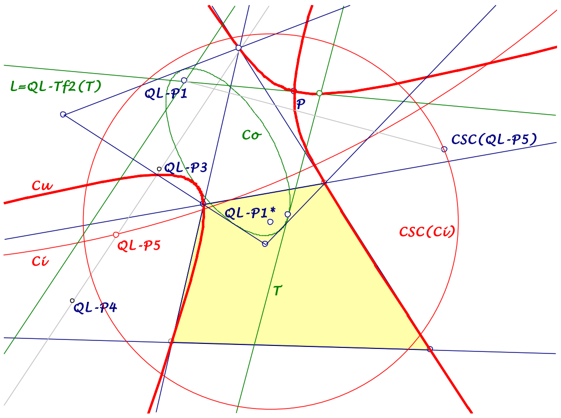
- The *QL-DT* inscribed conic *Co* is also the envelope of the *QL-Tf2*-images of lines through a point *P*, which is the *CSCe*-image of a line through *QL-P6* perpendicular to *QL-P3.QL-P4.QL-P5.QL-P6*.
- The *DT*-coordinates of point *P*:

$$\{m^2 n^2 L\ (c^2 M + b^2 N)\ (2\,a^2\ (b^2 + c^2)\ M N - a^4 N M + b^4 N L + c^4 L M),$$
$$n^2 l^2 M\ (a^2 N + c^2 L)\ (2\,b^2\ (c^2 + a^2)\ N L + a^4 N M - b^4 N L + c^4 L M),$$
$$l^2 m^2 N\ (b^2 L + a^2 M)\ (2\,c^2\ (a^2 + b^2)\ L M + a^4 N M + b^4 N L - c^4 L M)\}$$

- The conic *Co* is also the envelope of the polars of point *P* wrt inscribed conics of the quadrilateral.
- The intersections *Y* of lines *L* through *P* and their *QL-Tf2*-image give a conico-pivotal isocubic *Cu* (see B. Gibert: Special Isocubics, 8.1):
  … reference triangle: Ceva triangle of *P* wrt *QL-DT*,
  … isoconjugation with fixed point *P*,
  … pivotal conic *Co*.
  … If there are real tangents from *P* to *Co*, they are also tangent to the isocubic *Cu* and *P* is a nodal point.



- The isocubic *Cu* is also the locus for the contacts *X* and *Y* of tangents from *P* to inscribed conics of the quadrilateral; the isoconjugation of *Cu* swaps *X* and *Y*.



- History of point *QL-P5*: *QL-P5* is a point on *Ci*; the perpendicular bisector *T* of *QL-P1* and *CSC(QL-P5)* is tangent *Co; QL-Tf2(T)* is the line *L* = *P.QL-P1*; *CSC(L)* is a parallel of *QL-P3.QL-P4.QL-P5.QL-P6* through *QL-P1* and the intersection of *T* and *L* is a point on the cubic *Cu*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de