EQF-Note 2014-03-22

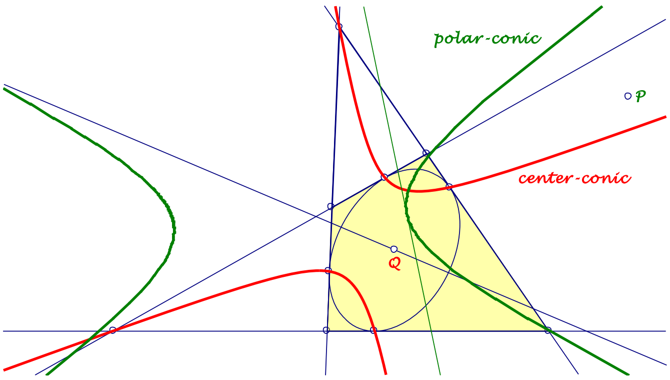
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Special Conics for QG, QL, QA

The inscribed conics of a quadrigon lead to other conics:

1. *The boundary points of an inscribed conic and the intersections of opposite sides give a new conic.*
2. *The polars of a point wrt inscribed conics envelope a further conic.*

These conics will be discussed for quadrigon, quadrilateral and quadrangle. – Barycentric DT-coordinates are used for a QL-quadrigon.



Center-Conics

Consider an inscribed conic of a quadrigon, centered in $Q(\alpha:\beta:\gamma)$ on the Newton Line *QL-L1* with the equation

$$\gamma(l^2\alpha+n^2\gamma)x^2-m^2\alpha\gamma y^2+\alpha(l^2\alpha+n^2\gamma)z^2=0.$$

This conic has the boundary points

$$(\pm lm\alpha:\pm(l^2\alpha+n^2\gamma):\pm mn\gamma).$$

In connection with the intersections of opposite sides they give a new conic with the equation

$$l^2(l^2\alpha+n^2\gamma)x^2-m^2(l^2\alpha-n^2\gamma)y^2-n^2(l^2\alpha+n^2\gamma)z^2=0$$

and the center

$$(m^2n^2(l^2\alpha-n^2\gamma):-n^2l^2(l^2\alpha+n^2\gamma):-l^2m^2(l^2\alpha-n^2\gamma)).$$

Such a conic here shall be named as **center-conic of *Q***, defined by the center $Q(\alpha:\beta:\gamma)$ of an inscribed conic.

Polar-Conics

Consider the polars of an arbitrary point $P(u:v:w)$ wrt inscribed conics of a quadrigon. These polars have the equation

$$\gamma u(l^2\alpha+n^2\gamma)x-\alpha\gamma m^2vy+\alpha w(l^2\alpha+n^2\gamma)z=0$$

and envelope a conic with the equation

$$l^4u^2x^2 + m^4v^2y^2 + n^4w^2z^2$$
$$-2l^2m^2uvxy - 2m^2n^2vwyz - 2n^2l^2wuzx = 0$$

and the center

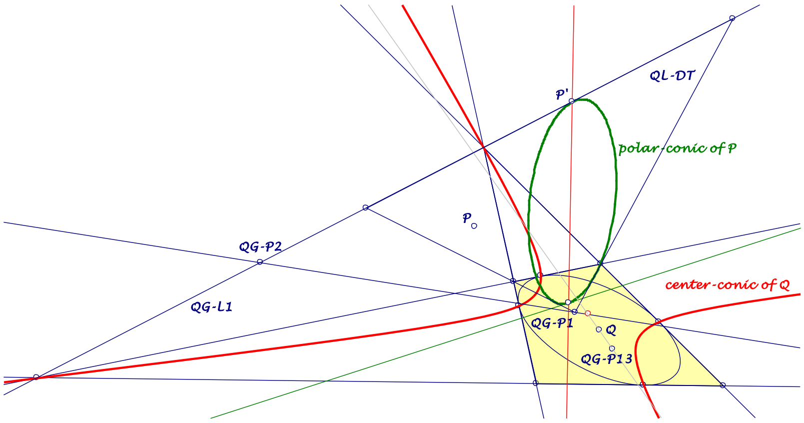
$$(m^2v + n^2w : n^2w + l^2u : l^2u + m^2v)$$

tangent to the polar in the point

$$(\alpha^2 l^2 m^2 vw : (l^2\alpha + n^2\gamma)^2 wu : m^2 n^2 uv\gamma^2)\,.$$

Such a conic here shall be named as **polar-conic of *P***, defined by the polars of a point $P(u:v:w)$ wrt inscribed conics.

Properties wrt a Quadrigon:

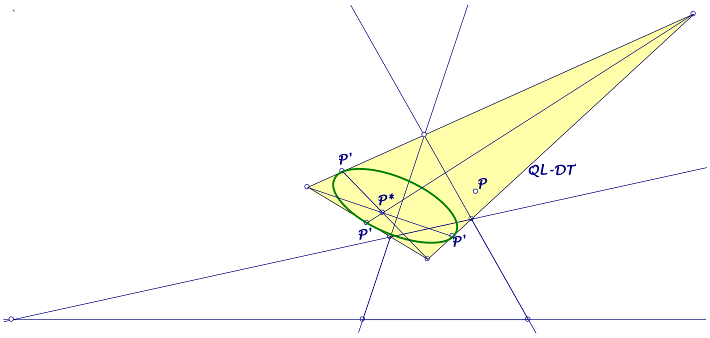


- The centers of center-conics are collinear on the line *QG-P1.QG-P2*.
- The polar of *P* wrt an inscribed conic and the polar of *P* wrt the corresponding center-conic intersect on the polar conic of *P*.
- The center *Q* of an inscribed conic and the center of the corresponding center-conic are collinear with *QG-P13*.
- The polars of a point $P(u:v:w)$ wrt center-conics have a common point $P'(n^2w:0:l^2u)$ on the third diagonal *QG-L1* of the quadrigon.
- The point *P´* is the intersection of *QG-L1* and the trilinear polar of *P* wrt the *QA*-Diagonal Triangle *QA-DT*.
- The point *P´* is the boundary point of the third diagonal and the polar-conic of *P*.
- The point *P´* is the common pole of *P.QG-P1* for center-conics.
- Points *P* on a line through *QG-P1* have the same *P´*:
  … for *QG-L2* the point *P´* is *QG-P3*;
  … for a parallel to *QG-L1* through *QG-P1* the point *P´* is *QG-P2*;
  … for *QG-P1.QG-P2* the point *P´* is he point at infinity of *QG-L1*;
  … for *QG-P1.QG-P3* the point *P´* is the intersection of *QG-L1* and *QG-L2*.

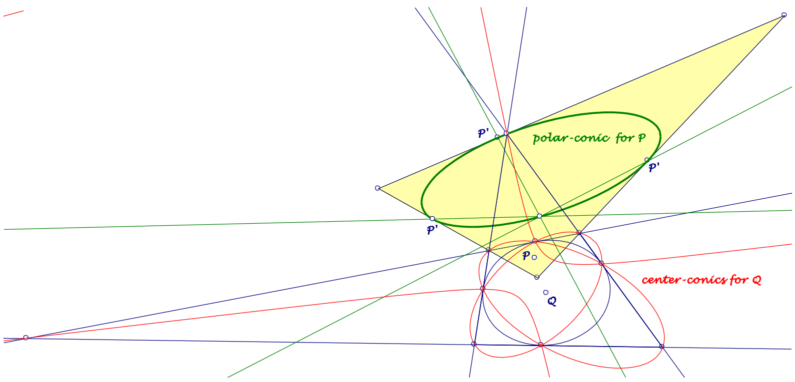
Properties wrt a Quadrilateral and its 3 QG-versions:

The polar-conic of a point *P* is an inscribed conic of the *QL*-Diagonal Triangle *QL-DT*.

- The polar-conic of a point *P* is the envelope of the *QL-Tf2*-images of lines through *P*.
- Let * be a *QL-DT*-isoconjugation with fixed points in the trilinear poles of the *QL*-lines. For a point *P* the polar-conic is a *QL-DT* inscribed conic with Brianchon point *P**. The boundary points are the three points *P´* wrt the 3 *QG*-versions of the quadrilateral.



- For *P* = *QL-P1* the polar conic is a *QL-DT* inscribed conic with asymptotes in the Steiner Axes (see *QL-Tf1*). This conic is detailed worked out in *EQF-Note 2014-03-17*.
- For *P* = *QL-P13* the polar conic is the *QL-DT* inscribed Steiner ellipse.
- For *P* = *QL-P8* the polar conic is the *QL-DT* inscribed conic with Brianchon point *QL-P13*.
- Let *G* be the Gergonne point of *QL-DT*, then
$P = G*((-a+b+c)m^2n^2 : (a-b+c)n^2l^2 : (a+b-c)l^2m^2)$
gives the incircle of *QL-DT* as polar-conic.



For a center *Q* of a *QL*-inscribed conic there are three center-conics wrt the three *QG*-versions of the quadrilateral.

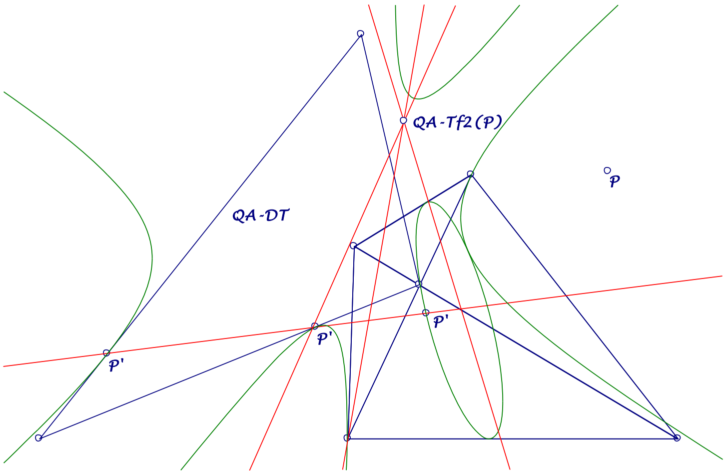
- The polars of a point *P* wrt such a trio of center-conics have a common point on the polar-conic of *P*.

- This point is the boundary point of the polar-conic and the polar of *P* wrt the inscribed *QL*-conic centered in *Q*.
- This point is the image of *P* wrt a *QL-DT*-isoconjugation with fixed points in the boundary points of the inscribed *QL*-conic centered in *Q*.

Properties wrt a Quadrangle and its 3 QG-versions:

For a point *P* there are three polar-conics wrt the three *QG*-versions of the quadrangle.

- The polars of *P* wrt its three polar-conics have a common point in the Involutary Conjugate *QA-Tf2* of *P*.



- The three points *P´* wrt the *QG*-versions of a quadrangle are collinear on the trilinear polar of *P* wrt the *QA*-Diagonal Triangle *QA-DT*.
- The three points *QL-P1´* for the *QG*-versions of a quadrangle are collinear on the trilinear polar of the Gergonne-Steiner Point *QA-P3* wrt *QA-DT*. This line is perpendicular *QA-P11.QA-P32* and contains *QA-P16*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de