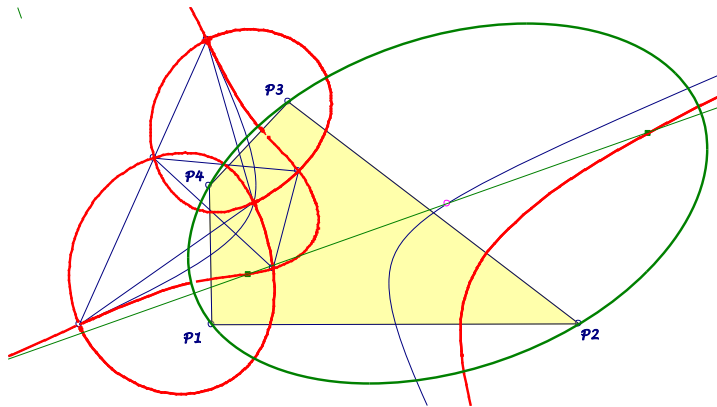


EQF-Note 2014-04-02

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Foci of QA-Circumscribed Conics

The foci of inscribed conics of a quadrilateral lie on the QL-Quasi Isogonal Cubic QL-Cu1. In this note the locus for the foci of circumscribed conics of a quadrangle is described. The locus is a sextic, built up of three isogonal invariant curves wrt the orthic triangle of the QA-diagonal triangle.



Let QA be a quadrangle, DT its diagonal triangle and OT the orthic triangle of DT .

1. The centers of QA -circumscribed conics lie on the Nine-point Conic $QA-Co1$ (see *EQF*).
2. The foci of QA -circumscribed conics are isogonal conjugated wrt OT .
3. The foci of a QA -circumscribed conic with center Z lie on an isogonal pivotal isocubic wrt the reference triangle OT and pivot Z .
4. The midpoints for the OT -pedal circles of the considered foci lie on $QA-Co1$.
(This property gives the possibility for construction and calculation.)
5. For $QA-DT$ as reference triangle the locus for the considered foci has the equation (DT -notation):

$$\begin{aligned}
& \frac{p^2 y z}{-SA x^2 + SB y (x + z) + SC z (x + y)} \\
+ & \frac{q^2 x z}{SA x (y + z) - SB y^2 + SC z (x + y)} \\
+ & \frac{r^2 x y}{SA x (y + z) + SB y (x + z) - SC z^2} = 0
\end{aligned}$$

6. This sextic for the considered foci consists of three *OT*-isogonal invariant curves, each containing two vertices of *DT* and the corresponding vertices of *OT*.
7. Construction: Let *U*, *V*, *W* be the vertices of *OT*.
- *C_i* circle with variable radius, centered in a point *P* on the conic *QA-Co1*,
 - *P_{u,v,w}* in each case one intersection of *C_i* and the sides *u=VW*, *v=WU*, *w=UV*,
 - *L_i* perpendicular in *P_i* wrt the corresponding side,
 - *Q_i* intersections of *L_j* and *L_k*,
 - *Co_i* conic as locus of *Q_i* wrt variable radius of *C_i*:
 - The common two points of *Co_u*, *Co_v*, *Co_w* are the searched foci for a *QA*-circumscribed conic centered in *P*.
8. Special points:
- the vertices of *DT* and *OT*,
 - the infinity points of *QA-Co1*,
 - the foci of the circumscribed parabolas *QA-2Co1* (on the *OT*-circumcircle as *OT*-isogonal conjugates of the infinity points of *QA-Co1*),
 - the reflections of the *OT*-vertices in the intersections of the corresponding mid-parallel of *OT* and *QA-Co1* (these six points lie on a conic, they are the partner-foci for the *OT*-vertices),
 - the foci of the *QA*-Orthogonal Hyperbola *QA-Co2* and the foci of the Gergonne-Steiner Conic *QA-Co3*,
 - the foci for the Circumscribed Harmonic Conics *QG-Co2* and the foci for the *M3D* Hyperbolas *QG-Co3* of the three quadrigon components.

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