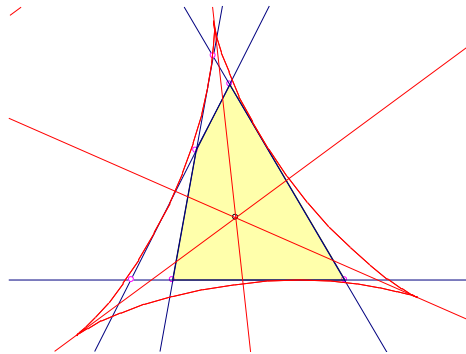


Background for these notes is:  
 Chris van Tienhoven: Encyclopedia of Quadri-Figures  
<http://chrisvantienhoven.nl/>

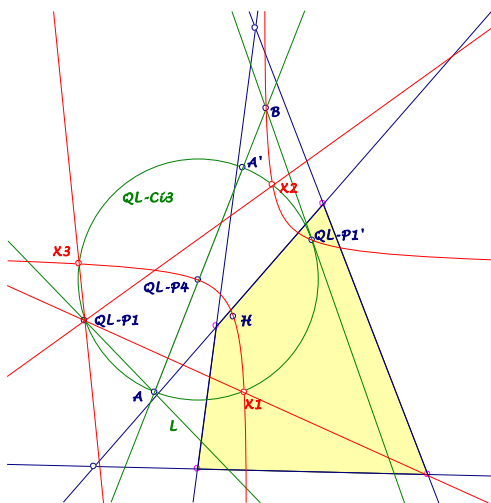
### Construction of Eckart's Cubic $QL-Cu2$

$QL-Cu2$  is the cubic for the centers of 27 cardioids tangent to four lines. The construction of  $QL-Cu2$  depends on the asymptotes, intersecting in the Miquel Point  $QL-P1$  with angles of  $60^\circ$ . They are parallel to the axes of the Kantor-Hervey Deltoid  $QL-Qu2$ , which is tangent to the lines of the quadrilateral and centered in  $QL-P3$ .



### Axes of the cubic $QL-Cu2$

There is a construction of the axes of  $QL-Cu2$  by Bernard Keizer in his paper "Trisection of the angle: the Chasles construction" (*QFG*-message 535):

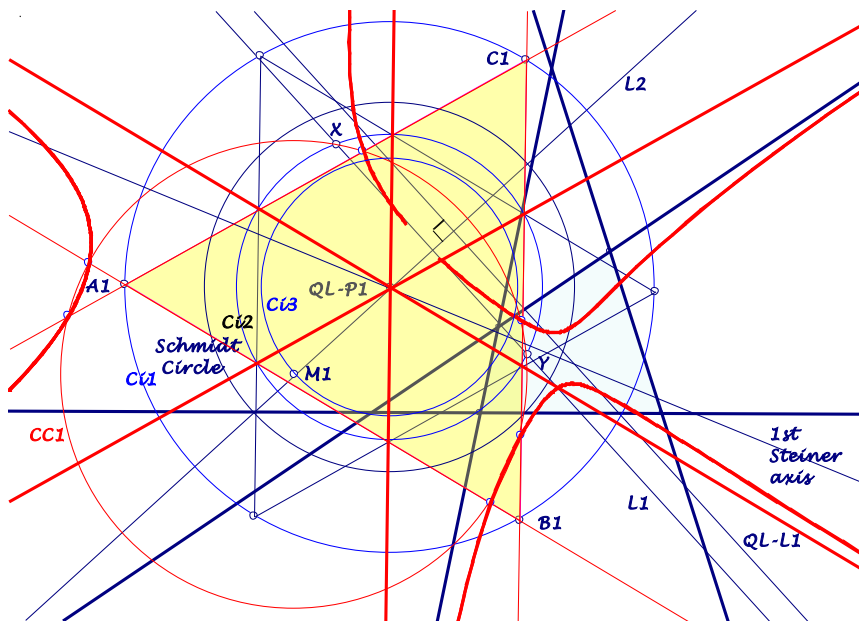


- $QL-Ci3$  Miquel Circle with midpoint  $QL-P4$ ;
- $QL-P1$  Miquel Point on  $QL-Ci3$  with antipode  $QL-P1'$ ;
- $L$  parallel to the Newton Line  $QL-L1$  through  $QL-P1$ ;
- A second intersection of  $L$  and  $QL-Ci3$ ;

- $H$  midpoint of  $A$  and  $QL-P1'$ ;
- $B$  intersection of  $A.QL-P4$  and the tangent in  $QL-P1'$  at  $QL-Ci3$ ;
- $H_y$  orthogonal hyperbola through  $QL-P4$ ,  $QL-P1'$ ,  $H$ ,  $B$ ;
- $X1, X2, X3$  further intersections of  $H_y$  and  $QL-Ci3$ ;
- $X_i.QL-P1$  are the asymptotes.

### Construction of $QL-Cu2$ with given asymptotes

This construction is valid for the McCay type and the Kjp type of  $QL-Cu2$  and also for quadrilaterals tangent to a circle.



- $Ci1$  variable circle round the Miquel Point  $QL-P1$ ;
- $A_1B_1C_1$  and  $A_2B_2C_2$  equilateral triangles, inscribed  $Ci1$ , with sides parallel to the given asymptotes;
- $Ci2$  circumcircle for the hexagon of the triangle intersections;
- $L1$  parallel to the Newton Line  $QL-L1$  in  $2/3$  distance wrt  $QL-P1$ ;
- $X, Y$  intersections of  $L1$  and  $Ci2$ ;
- $Ci3$  reflection of  $Ci1$  in the Schmidt Circle (see  $QL-Tf1$ );
- $M1$  and  $M2$  intersections of  $Ci3$  and a perpendicular line  $L2$  through  $QL-P1$  wrt  $QL-L1$ , so that  $M_i$  and two points of  $A_i, B_i, C_i$  lie on one side of the 1<sup>st</sup> Steiner Axis (see  $QL-Tf1$ );
- $CC_i$  circles round  $M_i$  through  $X$  and  $Y$ .
- The intersections of  $CC_i$  and the sidelines of  $A_iB_iC_i$  are points of  $QL-Cu2$ .