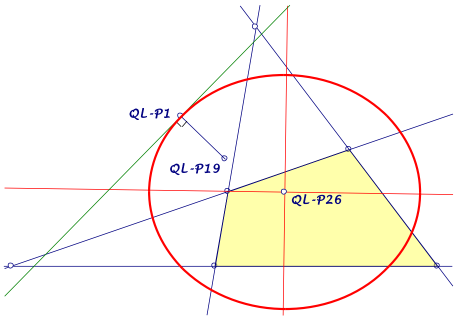
EQF-Note 2014-06-04

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Constant sum of QL-Distance-Squares

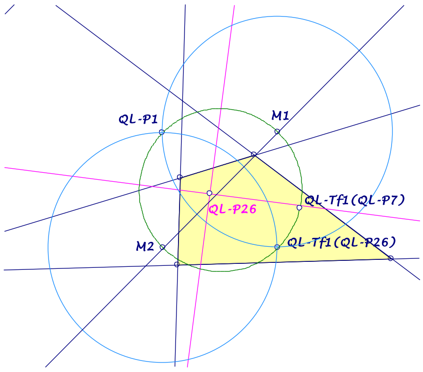
For the Least Squares Point QL-P26 the sum of the squares of the distances to the four lines of a quadrilateral is minimal. The loci for points with a constant sum of QL-distance-squares are homothetic ellipses with parallel axes, centered in QL-P26. – Calculations with barycentric coordinates are possible, but extensive and here omitted.



Let $d_i(P)$ be the distance of a point P to the line L_i of the quadrilateral, then we consider points X with

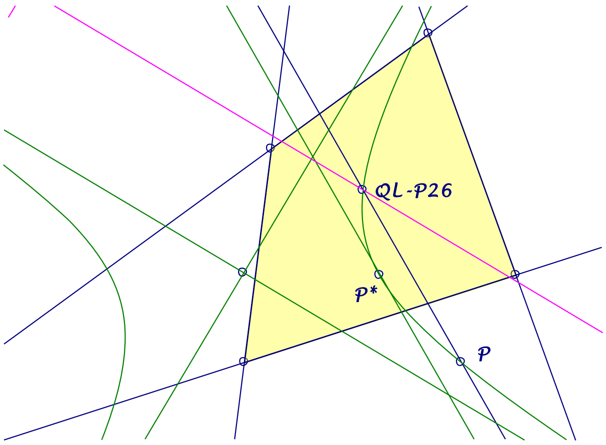
$$\sum_i d_i(P)^2 = \sum_i d_i(X)^2.$$

- The loci of these points are ellipses *E(P)*, centered in the Least Square Point *QL-P26* (which is the centroid of its pedal quadrangle).
- If *P* is the Miquel Point *QL-P1*, the tangent in *QL-P1* to *E(P)* is perpendicular to *QL-P1.QL-P19* (*QL-P19* is the centroid of the collinear degenerated pedal quadrangle of *QL-P1*).
- In general: The tangent in *P* to *E(P)* is perpendicular to the line, connecting *P* and the centroid *P** of its pedal quadrangle.
- The ellipses *E(P)* are homothetic wrt *QL-P26*.
- The axes of *E(P)*: The circumcircle of *QL-P1*, *QL-Tf1(QL-P26)* and *QL-Tf1(QL-P7)* intersects the perpendicular bisector of *QL-P1.QL-Tf1(QL-P26)* in M_1 and M_2. The *QL-Tf1*-images of the circles round M_1 and M_2 through *QL-P1* are the searched orthogonal axes through *QL-P26* (see also *EQF-Note 2014-05-30* in *QFG*-message 583).



With these properties it is possible to construct the ellipses *E(P)*.

- The axes of *E(P)* are the loci of points *Q*, so that *Q*, the centroid *Q** of the pedal quadrangle of *Q* and *QL-P26* are collinear.
- The axes of *E(P)* are parallel to the asymptotes of all orthogonal hyperbolas, constructed with *QL-P26*, an arbitrary point *P*, the centroid *P** of the pedal quadrangle of *P* with a tangent in *P** parallel *P.QL-P26*.



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