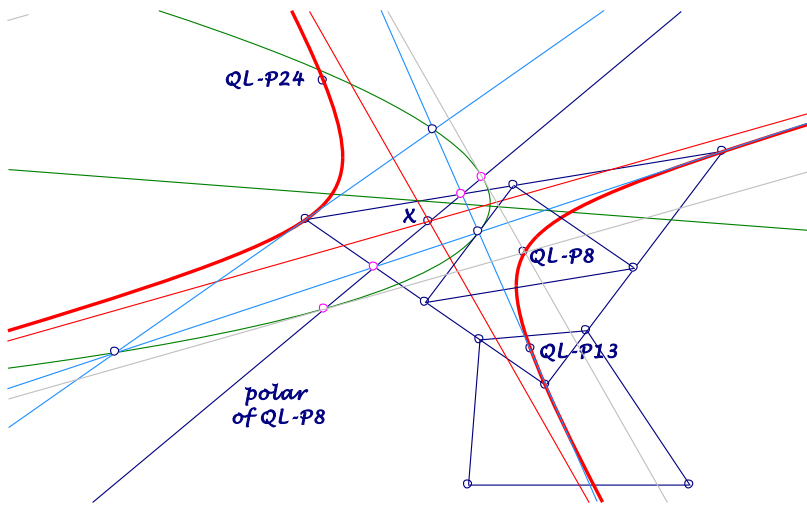


Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

QL-Hyperbola wrt QL-P8

A special QL-DT-isoconjugation maps the polar of QL-P8 wrt QL-Co1 into a QL-DT-circumscribed hyperbola through QL-P8, QL-P13 and QL-P24. The geometry of the isoconjugation and the hyperbola is worked out. – Reference triangle for barycentric coordinates is QL-DT.



Preliminary remarks

- The inscribed parabola $QL-Co1$ of a quadrilateral is not only tangent to the lines of the quadrilateral, but also to the sidelines of the medial triangle of $QL-DT$.
- $QL-Co1$ has the equation

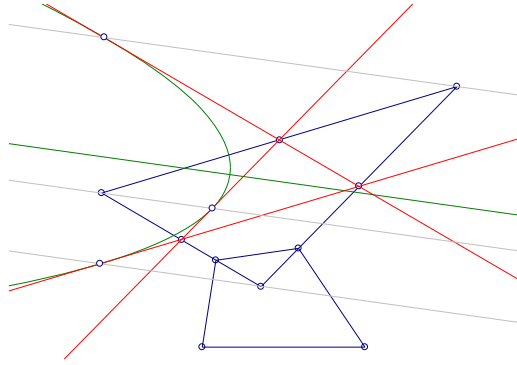
$$(l^2 - m^2)(l^2 - n^2)x^2 + (m^2 - n^2)(m^2 - l^2)y^2 + (n^2 - l^2)(n^2 - m^2)z^2 = 0.$$

The points of tangency wrt the medial triangle of $QL-DT$ are

$$(\pm(m^2 - n^2) : \pm(n^2 - l^2) : \pm(l^2 - m^2))$$

without the point with coordinates of equal sign, which is the point of infinity of $QL-Co1$.

- The points of tangency are the intersections of the three QG-Centroids Lines $QG-L3$.
- The points of tangency are the intersections of $QL-Co1$ with parallels to $QL-L1$ through the vertices of $QL-DT$.



The QL-DT-isoconjugation

- We consider an isoconjugation wrt *QL-DT* with fixed points in the points of tangency and the point at infinity of *QL-Co1* as described above:

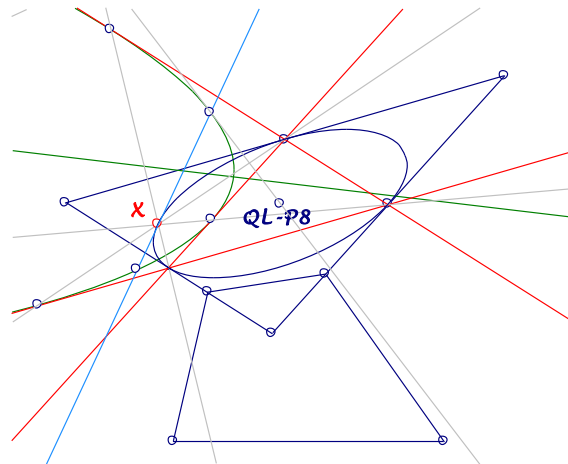
$$(x : y : z) \rightarrow ((m^2 - n^2)^2 yz : (n^2 - l^2)^2 zx : (l^2 - m^2)^2 xy) .$$

- This isoconjugation maps the Miquel Point *QL-P1* in the intersection of *QL-L2* and *QL-P1.QL-P10*.
- The pole *X* of the isoconjugation (image of *QL-P8*)

$$X((m^2 - n^2)^2 : (n^2 - l^2)^2 : (l^2 - m^2)^2)$$
is the Brianchon point of *QL-Co1* wrt the *QL-DT*-medial triangle.

- The point *X* lies on the inscribed Steiner ellipse of *QL-DT*.
- The tangent at *X* to the Steiner ellipse is the polar of *QL-P8* wrt *QL-Co1* with the equation

$$\frac{x}{m^2 - n^2} + \frac{y}{n^2 - l^2} + \frac{z}{l^2 - m^2} = 0 .$$



The QL-DT circumscribed hyperbola

- The isoconjugation maps the polar of *QL-P8* wrt *QL-Co1* in a circumscribed hyperbola of *QL-DT* with the equation

$$(l^2 - m^2)xy + (m^2 - n^2)yz + (n^2 - l^2)zx = 0 .$$

- The hyperbola contains the points $QL-P8$, $QL-P13$, $QL-P24$.
- The hyperbola is centered in the Brianchon point X described above (see first figure).
- The asymptotes are parallel to the tangents through $QL-P8$ wrt $QL-Co1$.
- The tangents to the hyperbola in the $QL-DT$ -vertices are the QG -Centroids Lines $QG-L3$ of the three quadrigon versions.
- The $QL-DT$ -isotomic image of the hyperbola is its tangent in $QL-P8$.
- The tangent in $QL-P8$ at the hyperbola cuts the asymptotes on the $QL-DT$ -inscribed Steiner ellipse.

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