**EQF-Note 2014-08-18**

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

## A new aspect of QL-Cu1

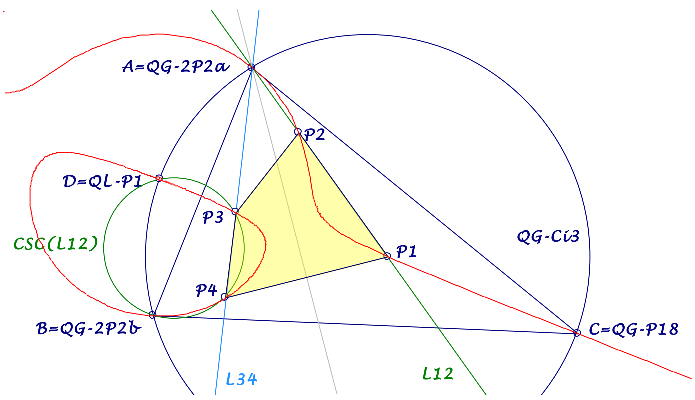
*Here the QL-Quasi Isogonal Cubic QL-Cu1 is considered wrt the Quasi Isogonal Triangle QG-Tr3 and the Miquel Point QL-P1. Background is the following property:*

***QL-Cu1* is the locus for the vertices of quadrigons with the same triangle *QG-Tr3* and the same point *QL-P1*.**

The cubic *QL-Cu1* of a quadrigon $P_1P_2P_3P_4$ has a clearly arranged equation, taking *QG-Tr3* as reference triangle for barycentric coordinates (see also *QFG*-message 657) and giving $P_4$ the coordinates $u$, $v$, $w$:

$$(c^2y^2+2S_Ayz+b^2z^2)(c^2u^2+2S_Bwu+a^2w^2)vx$$
$$-(c^2v^2+2S_Avw+b^2w^2)(c^2x^2+2S_Bzx+a^2z^2)uy=0$$

So it is obvious, to study the cubic wrt the reference triangle *QG-Tr3* and the Miquel Point *QL-P1*, which is a point on the circumcircle. *QL-Cu1* is isogonal invariant wrt *QG-Tr3* (see *EQF*).




Construction of quadrigons $P_1P_2P_3P_4$ with the same reference triangle *QG-Tr3* and the same *QL-P1*:

… let *ABC* be the reference triangle (later *QG-Tr3* with *A*,*B* = *QG-2P2a*,*b* and *C* = *QG-P18*),

... let *D* (later *QL-P1*) be a point on the circumcircle of *ABC*,

… then the later *CSC*-transformation (*QL-Tf1*) is defined by *D* and *A*, *B* (reflection in the angle bisector of

$\angle ADB$ and inversion wrt a circle round $D$ with radius $\sqrt{DA \cdot DB}$),

… let $L_{12}$ be an arbitrary line through $A$ and $L_{34}$ the reflection in the angle bisector of *QG-Tr3* at $A$,

… let *Ci* be the circle, which is the *CSC*-image of $L_{12}$,

… then the intersections of *Ci* and $L_{34}$ give the vertices $P_3$ and $P_4$ and their isogonal conjugates are $P_1$ and $P_2$.

Varying the line $L_{12}$ through $A$, the vertices of the quadrigons give *QL-Cu1*. This gives a further construction of *QL-Cu1*.

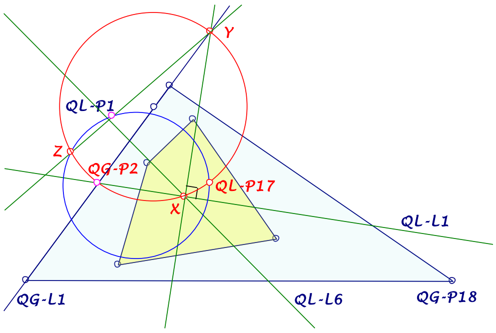
Considering the pencil of quadrigons with the same *QG-Tr3* and *QL-P1* we can study loci of *QL*-points. Only some examples:

… The Newton Line *QL-L1* and *QG-P1.QG-P17* are fixed lines.

… The locus of *QL-P4* is the tangent in *QL-P1* for *QL-Cu1*.

… The locus of *QL-P11* is the perpendicular bisector of *QL-P1.QG-P17*.

… The locus of *QA-P4* is a circle: *CSC*-image of *QG-P1.QG-P17*.




… The locus of *QL-P17* is a circle centered on *QG-L1* through

… *QL-P17*, *QG-P2*,

… the intersection $X$ of *QL-L1* and *QL-L6*,

… the intersection $Y$ of *QG-L1* and a perpendicular line to *QL-L1* through $X$,

… the second intersection $Z$ of *QL-Ci6* and *Y.QL-P1*.

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