EQF-Note 2014-09-06

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Remarkable QL-Triangle

The QL-points QL-P2, QL-P6, QL-P12 are collinear in Euler constellation. Here the geometry of a triangle with this Euler line is researched. Vertices are the common points of the three versions of the Nine-Point Conic QA-Co1 for a quadrilateral.
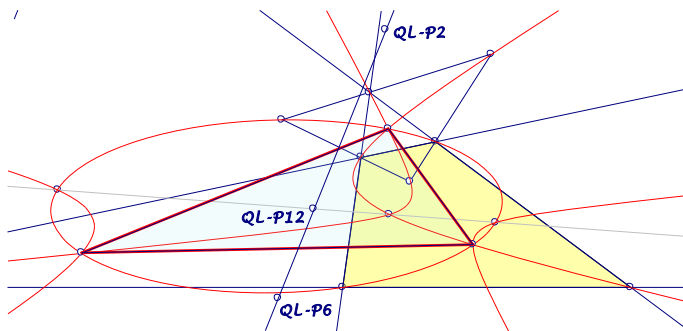

For a quadrilateral a version of *QA-Co1* contains two opposite *QL*-points, two midpoints of diagonals and one vertice of *QL-DT*. The double intersections of the three *QA-Co1* versions are the midpoints of the diagonals on the Newton Line *QL-L1*. The triple intersections give the vertices of a triangle *Tr*, which shall be tested here.

- **The Euler line of *Tr* is *QL-P2.QL-P6.QL-P12* with**
  centroid *QL-P2*,
  circumcenter *QL-P6*,
  orthocenter *QL-P12*.

- **The circumcircle of *Tr* is the Dimidium Circle *QL-Ci6*.**
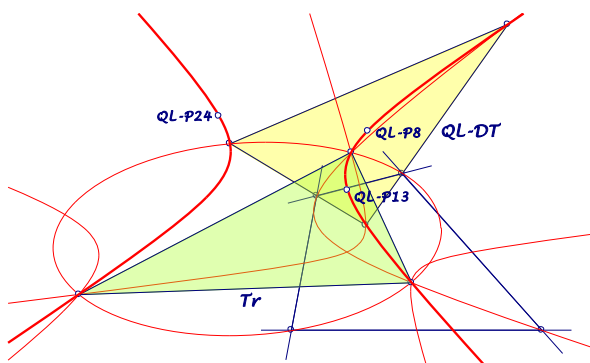
- **The triangles *Tr* and *QL-DT* have a common circumconic *Co* containing *QL-P8, QL-P13, QL-P24*.**

- There are two special points *X* and *Y* on the *Tr*-Steiner ellipse:
  … ***point X***: *Tr*-trilinear pole of *QL-L1* and contact point of *QL-L9* and the *Tr*-Steiner ellipse,
  … ***point Y***: *Tr*-isotomic conjugate of the point at infinity of *QL-L1* (diametral to point *X* on the *Tr*-Steiner ellipse).
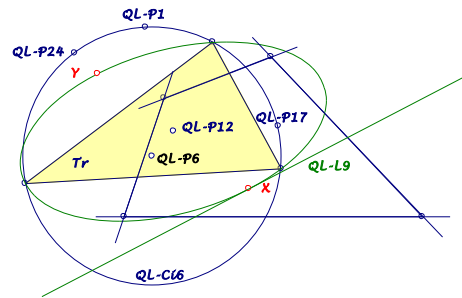
- **The sidelines of *Tr* are tangent to the Inscribed Parabola *QL-Co1* of the reference quadrilateral.**

- Brianchon point of *QL-Co1* wrt *Tr* is the point *Y*.
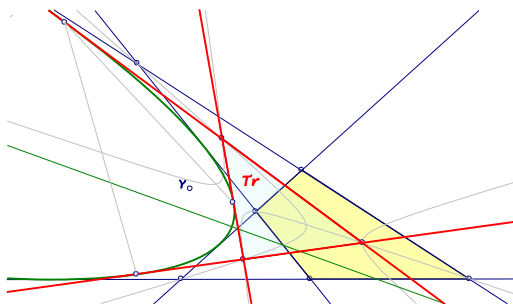
Trilinear poles and polars wrt *Tr*:

- The *Tr*-trilinear pole of *XY* is the *Tr*-isotom conjugate of the point at infinity of *QL-L9*.

- The *Tr*-trilinear poles of the *QL-DT*-sidelines are collinear on *XY*.

- The *Tr*-trilinear polars of the *QL-DT*-vertices have a common point *U*, which is the same for all points on *Co*.

- The *Tr*-trilinear poles of the *QL*-lines are collinear on the *Tr*-trilinear polar of *Y*.

- The *Tr*-trilinear polars of the *QL*-points give six lines, defining a quadrangle: The *Tr*-trilinear polars of the four quadrangle points have a common point *V* on *QL-L1*.

In the following properties let ° be an arbitrary isoconjugation wrt *Tr* (such as isogonal or isotomic conjugation):

- The *Tr*-isoconjugates of the *QL-DT*-vertices are collinear on the line
  $$L_1 = QL\text{-}P8°.QL\text{-}P13°.QL\text{-}P24°.$$

- The *Tr*-isoconjugates of the *QA-Co1*-versions are three lines
  … with *Tr*-trilinear poles on a line L_2,
  … giving a triangle Δ perspective *QL-DT,*
  … with center of perspectivity on *Co* in the *Tr*-isoconjugate of the intersection of *QL-L1* and L_1.

- The *Tr*-isoconjugates of the vertices of a *QL*-triangle component are collinear:
  … the four resulting lines give a quadrilateral with diagonal triangle Δ,
  … the *Tr*-trilinear poles of the four lines are collinear on a line L_3 with *Tr*-trilinear pole on L_2 and its isoconjugate is point *V* on *QL-L1* (independent of the isoconjugation).

- The three lines L_1, L_2, L_3 have a common point, dependent on the isoconjugation. Its isoconjugate is the 2nd intersection *Z* of *XY* and *Co,* independent of the isoconjugation.

There are special properties for *Tr*-isogonal and *Tr*-isotomic conjugations:

- Examples of *Tr*-isogonal conjugates *:
  … *QL-P1** is the point at infinity of *QL-L1*,
  … *QL-P17** is the point at infinity of *QL-L9*,
  … *QL-P24** is the point at infinity of L_1.

- Examples of *Tr*-isotomic conjugates ^:
  … *X^* is the point at infinity of *U.QL-P12*,
  … *Y^* is the point at infinity of *QL-L1*.

- Simson lines *Sl* for points *P* on *QL-Ci6* wrt *Tr*:
  … In general: *Sl(P)* contains the midpoint of *P.QL-P2* and is parallel *Q.QL-P1* with *Q* as 2nd intersection of *QL-Ci6* and a line through *P* perpendicular *QL-L1*.
  Special:
  … *Sl(QL-P1)* is *QL-L3*,
  … *Sl(QL-P17)* − also the Simson line of *QL-P17* wrt *QL-DT* − is parallel *QL-L5*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de