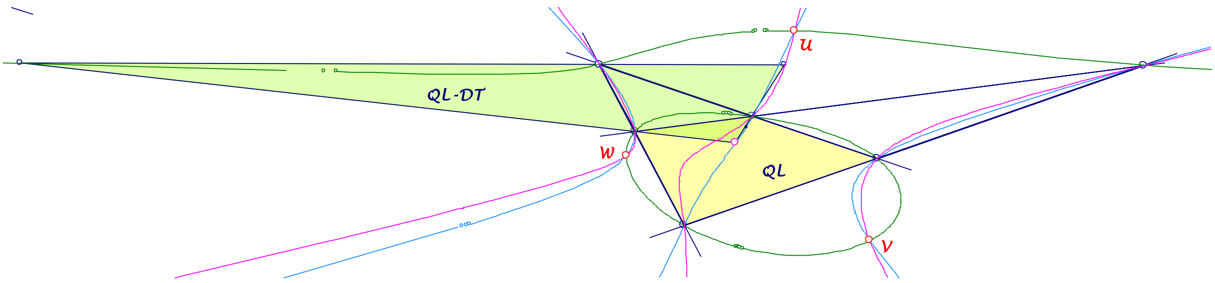
EQF-Note 2014-09-14

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Three QG-Cubics for a QL and a QA

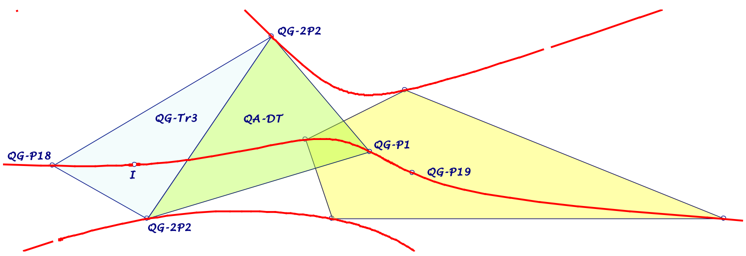
In EQF-Note 2013-01-10 (see my homepage) a pivotal isogonal cubic for the reference triangle QG-Tr3 with pivot QG-P1 is described. The three versions of this cubic for a quadrilateral give three common points, whose trilinear polars wrt QL-DT have a common point. For a quadrangle there are six double intersections on a conic.



The QG-cubic

- This *QG*-cubic is the pivotal isogonal cubic wrt *QG-Tr3* and pivot *QG-P1*.

- This *QG*-cubic is also an isocubic wrt *QA-DT*: isoconjugation is *QA-Tf2* and pivot *QG-P18*.

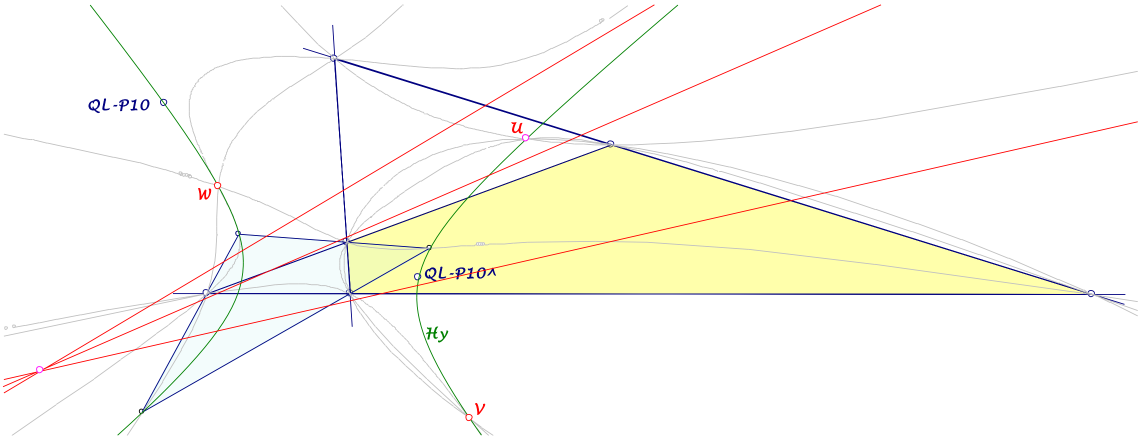


- This *QG*-cubic containes …
  … the vertices of the quadrigon,
  … the vertices of *QG-Tr3*,
  … the vertices of *QA-DT*,
  … the point *QG-P19*,
  … the in- and excenters of *QG-Tr3*.

- Tangents in the vertices of the quadrigon intersect in *QG-P18*.

The QG-cubics for a quadrilateral

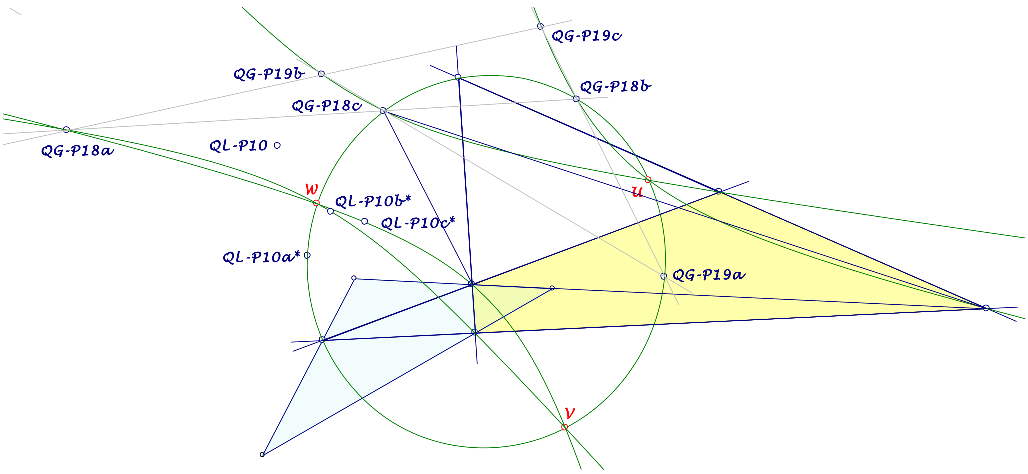
- The three *QG*-cubics for a quadrilateral have three common points *U*, *V*, *W* (beside the points of the quadrilateral).



- The common points *U*, *V*, *W* lie on an orthogonal hyperbola *Hy* …
  …circumscribed *QL-DT*
  …through *QL-P10* and its image *QL-P10^* wrt an *QL-DT*-isoconjugation with fixed points in the *QL-DT*-trilinear poles of the *QL*-lines.

These common points can be constructed as intersections of conics:

- The three *QG-P18* points lie collinear (see *EQF*) on the sidelines of the *QG-P19*-triangle.



- The common points *U*, *V*, *W* lie on a conic *Co* …
  …through *QG-2P2*, *QG-P19* of one *QG*-version
  …and *QG-P18* of the two other *QG*-versions
  …(and the isogonal conjugate *QL-P10** wrt *QG-Tr3* of the 1st *QG*-version).

The intersections of these three conics give the common points *U*, *V*, *W*, which not always will be real.

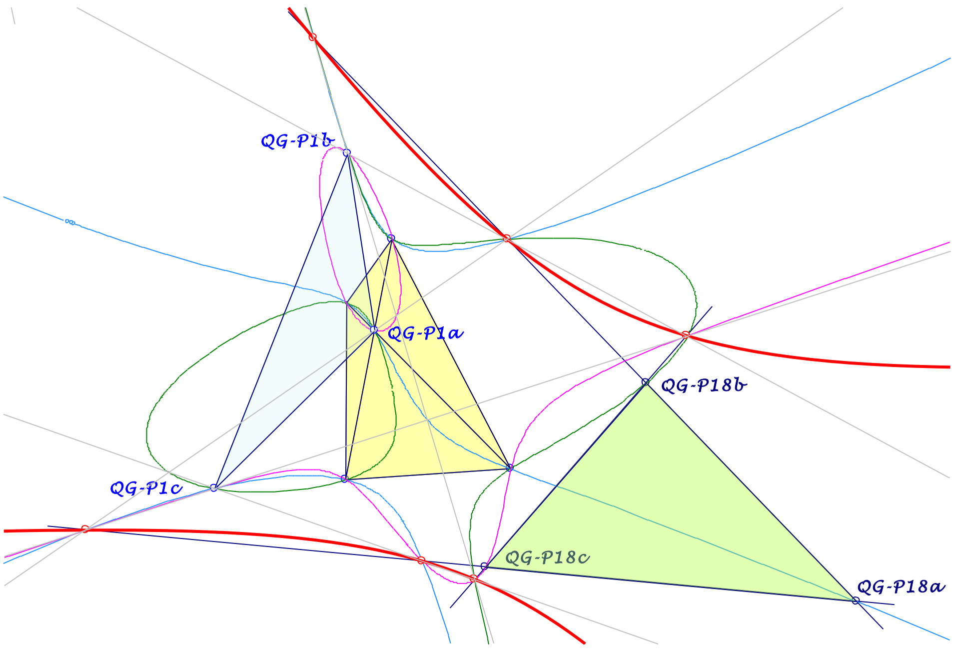
- The *QL-DT*-trilinear polars of *U*, *V*, *W* have a common point with *DT*-coordinates $(S_A(m^2S_C^2 - n^2S_B^2) : S_B(n^2S_A^2 - l^2S_C^2) : S_C(l^2S_B^2 - m^2S_A^2))$.

- The *QL-DT*-trilinear poles of lines through this common point give the orthogonal hyperbola *Hy*.

The QG-cubics for a quadrangle

- The common points of the three *QG*-cubics for a quadrangle are the *QA*-vertices and the *QA-DT*-vertices.

- The double intersections of the cubics lie on the sidelines of the *QG-P18*-triangle as *QA-Tf2*-partners.



- Construction of the double intersections: Intersections of the line *QG-P18a.b* with the *QA-DT* circumconic through *QG-P19a,b*, …

- The six double intersections lie on a conic.

- There are further collinear relations indicated in the figure.

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