EQF-Note 2014-10-04

### Steiner Axes, Newton Line and QL-Cu1

The intersections of the Steiner axes and the Newton Line QL-L1 are tested wrt the QL-Quasi Isogonal Cubic QL-Cu1.
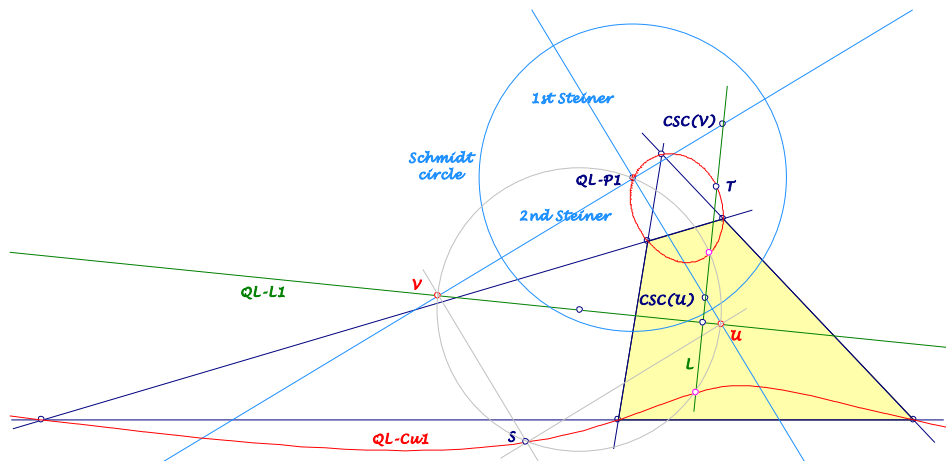
Preliminary remarks: The Steiner axes and the Schmidt circle are described in *EQF* wrt the transformation *QL-Tf1* (shortened *CSC*). Some points on the cubic *QL-Cu1* are already in *EQF*: beside *QL-P1* the point *S* as intersection of *QL-Cu1* and its asymptote and point *T = CSC(S)* on a line *L* through *QL-L1* ∩ *QL-L6* perpendicular *QL-L1*. The point *QL-L1* ∩ *QL-L6* is the center of a *QL*-inscribed conic with foci F_i, which are in the unipartite case of *QL-Cu1* the *EQF*-points *QL-2P2*.

Let *U* and *V* be the intersections of the 1st and 2nd Steiner axis with the Newton line *QL-L1*.

- *U* and *V* lie on a circle round *QL-L1* ∩ *QL-P1.QL-P4* through *QL-P1*.

- This circle contains in the bipartite case also the foci F_i collinear with *T* on *L*.

- The 4th rectangle point of *U, QL-P1* and *V* is *S*.

- A circle round *V* through the *CSC*-fixed points on the 1st Steiner axis cuts the 2nd Steiner axis on *QL-Cu1* in points with tangents parallel to the asymptote (parallel *QL-L1*).
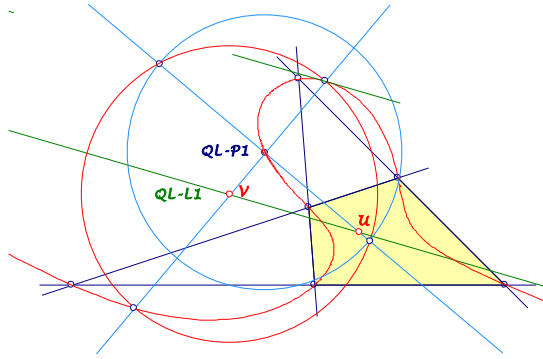
- The *CSC*-images of *U* and *V* are the intersections of the Steiner axes and the line *L* (which is the *CSC*-image of the circle above).

The unipartite case of QL-Cu1

- If *QL-Cu1* is unipartite, the point *U* is an inner point of the Schmidt circle.
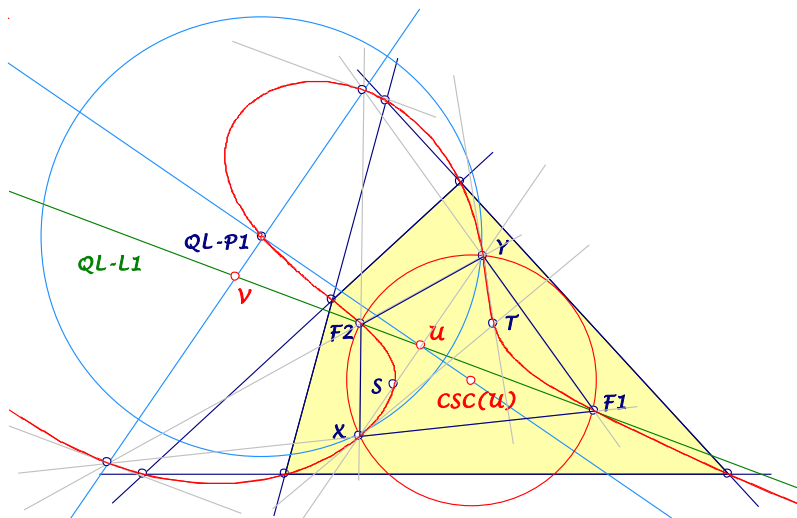
- A circle round *CSC(U)* orthogonal wrt the Schmidt circle intersects *QL-L1* on *QL-Cu1* in the foci $F_i = QL-2P2$.

- This circle is *CSC*-invariant; *CSC*-partners on the circle lie collinear with *U*.

- This circle has two further intersections *X* and *Y* with *QL-Cu1*: these are the intersections of the line *US* and the Schmidt circle, *CSC*-partners with common tangential *T*.

- The lines, connecting F_i with *X, Y* intersect with the 2nd Steiner axis on *QL-Cu1* in points with a tangent parallel to the asymptote.

- A circle round *CSC(V)* through the *CSC*-fixed points *QL-2P3* contains the points F_i. This circle is also *CSC*-invariant; *CSC*-partners on the circle lie collinear with *V*.

The bipartite case of QL-P1

- If *QL-Cu1* is bipartite, the point *U* is an outer point of the Schmidt circle.

- A circle round *U* orthogonal wrt the Schmidt circle intersects the line *L* on *QL-Cu1* in the foci F_i.

- The circle is *CSC*-invariant; *CSC*-partners on the circle lie collinear with *CSC(U)*.
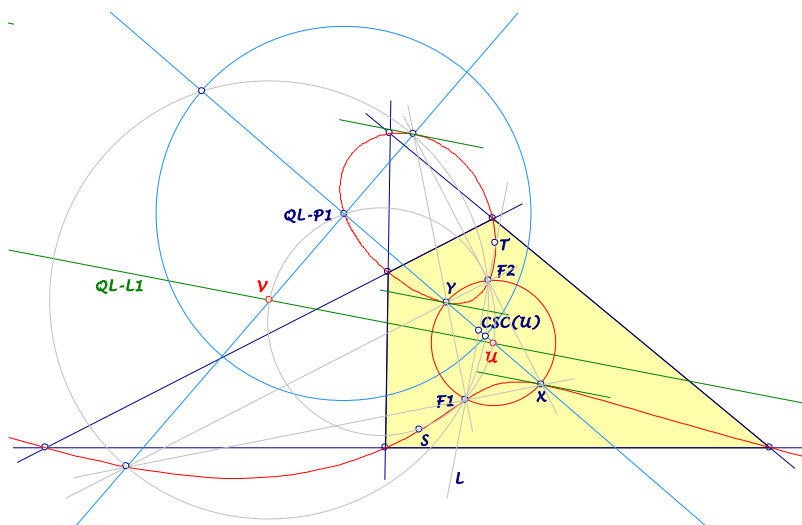

- The foci F_i are also points of the circumcircle of *U, V, QL-P1* (see above) and have the common tangential *S*.

- The circle has two further intersections *X* and *Y* with *QL-Cu1* on the 1st Steiner axis, *CSC*-partners with tangents parallel to the asymptote.

- The lines, connecting the foci F_i with *X, Y* intersect the 2nd Steiner axis on *QL-Cu1* in *CSC*-partners with tangents parallel to the asymptote.

- A circle round *V* through the *CSC*-fixed points *QL-2P3* contains the foci F_i. This circle is also *CSC*-invariant; *CSC*-partners on the circle are collinear with *CSC(V)*.

Final remark

The circle round *U* in the bipartite case of *QL-Cu1* or round *CSC(U)* in the unipartite case, which is orthogonal to the Schmidt circle, intersects *QL-Cu1* in four points: the foci F_i and

the points *X, Y* (as described above). The cyclic quadrigon F_1XF_2Y and the reference quadrilateral have the same Miquel Point *QL-P1*, the same Newton Line *QL-L1*, the same Steiner axes, the same Schmidt circle and the same cubic *QL-Cu1*. Background:

- **Every circle orthogonal to the Schmidt circle and centered on the 1st Steiner axis gives two pairs of *CSC*-partners on *QL-Cu1* as opposite vertices of a cyclic quadrigon. Opposite sides of this quadrigon intersect with the 2nd Steiner axis on *QL-Cu1* in points with tangents parallel to the asymptote.**
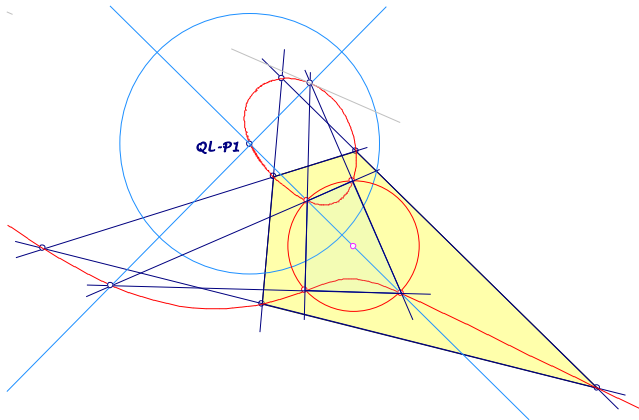
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