EQF-Note 2014-10-21

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

The CSC-Transformation for Pentalaterals

The CSC-images of a point wrt the 5 quadrilaterals of a Pentalateral are concyclic. The midpoint of the circle shall be the image of the point wrt the 5L-CSC-transformation. The geometry of this − not involutary − transformation here will be tested with CABRI.
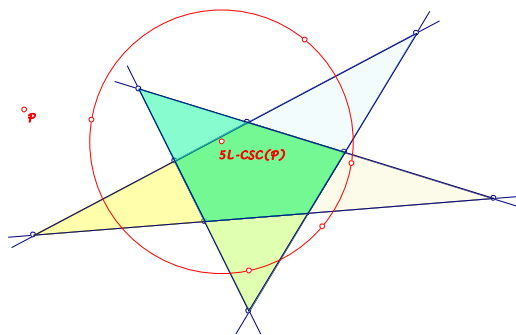
Some *5L*-points will be of importance for the geometry of this transformation (see *QFG*-message 710):
… The Miquel points of the 5 *QL*-components of a *5L* are concyclic on the *QL-P1*-circle.
… *5L-P1* is the center of the *QL-P1*-circle.
… *5L-P4* is the center of the inscribed conic.
… F_1 and F_2 are the foci of the inscribed conic and $F_1°$ and $F_2°$ their inverses wrt the *QL-P1*-circle.
… *5L-Px* shall be the intersection of $F_1°.F_2$ and $F_1.F_2°$.
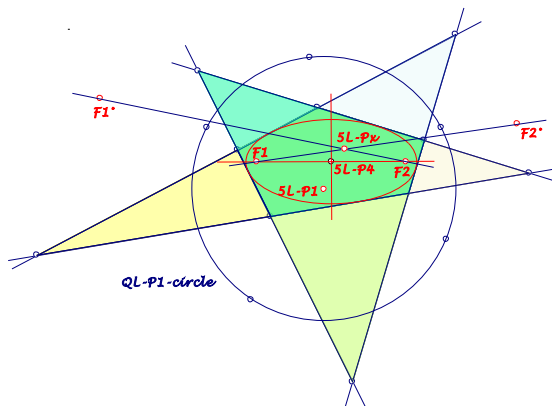
Definition:
5L-CSC(P) is the center of the 5 concyclic *CSC*-images of *P*.
The *CSC*-circle of *P* is the corresponding circle.
(wrt *CSC* see *EQF* under *QL-Tf1*)

Properties:

***5L-CSC*-images:**

1. *5L-CSC* is not involuntary.

2. *5L-CSC* swaps F_1 and F_2.

3. The *5L-CSC*-image of $L_i \cap L_j$ is the circumcenter $O_{i,j}$ for the triangle of the remaining three *5L*-lines.

4. The *5L-CSC*-image of *5L-P1* is the reflection in *5L-P4*.

5. The *5L-CSC*-image of *5L-Px* is *5L-P1*.

6. The *5L-CSC*-image of the *QL-P1*-circle is the line at infinity.

7. The image of a *QL-P1*-point is the point at infinity of a line, perpendicular to the remaining *5L*-Line.

8. For two points inverse wrt the *QL-P1*-circle the *5L-CSC*-images are symmetric wrt *5L-P4*.

9. The *5L-CSC*-image of a line is a conic through *5L-P1*.

10. The *5L-CSC*-image of a *5L*-line L_i is a conic
    ... through *5L-P1*,
    …the center of $CSC(L_i)$ wrt the corresponding *QL*,
    …the four circumcenters $O_{i,j}$ (see 3).
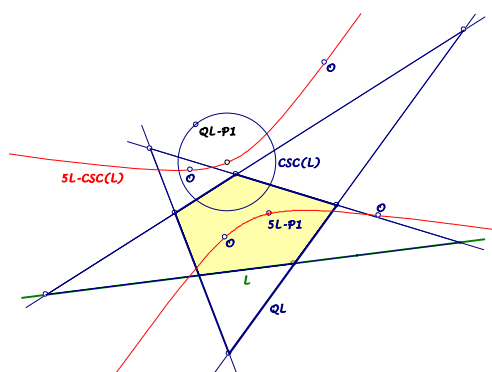
11. The *5L-CSC*-images of tangents at the *QL-P1*-circle are parabolas through *5L-P1*.

12. For lines through *5L-Px* the *5L-CSC*-image degenerates to a line through *5L-P1*.

13. Examples: $F_1.F_2{}^\circ \rightarrow F_2.F_2{}^\circ$, $F_2.F_1{}^\circ \rightarrow F_1.F_1{}^\circ$,
   *5L-P4.5L-Px* → *5L-P1.5L-Px* → *5L-P1.5L-P4*,

14. Lines through *5L-Px* and their *5L-CSC*-line intersect on an orthogonal hyperbola *Hy* through *5L-P1, 5L-Px, F1°* and *F2°* , centered in the midpoint of $F_1°.F_2°$ and tangent to *5L-P1.5L-P4*.
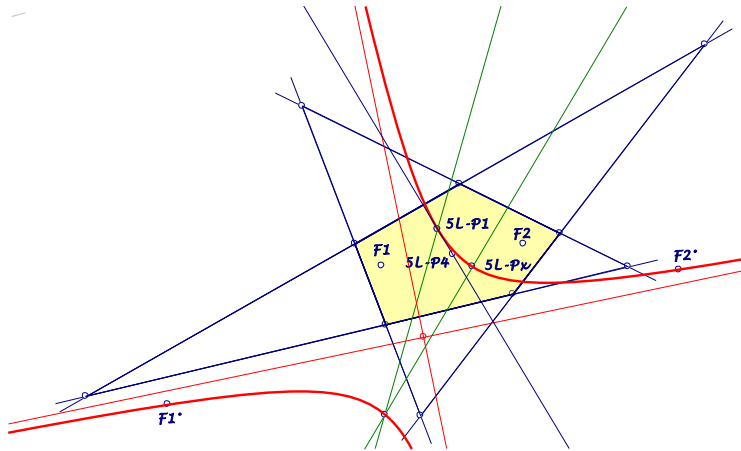

15. The *5L-CSC*-image of a circle is a conic.

CSC-circles:

16. The *CSC*-circle of an intersection of two *5L*-lines is the circumcircle of the triangle of the remaining three *5L*-lines.

17. The *CSC*-circles of points on the *QL-P1*-circle degenerate to lines tangent to the inscribed conic of the *5-L*.
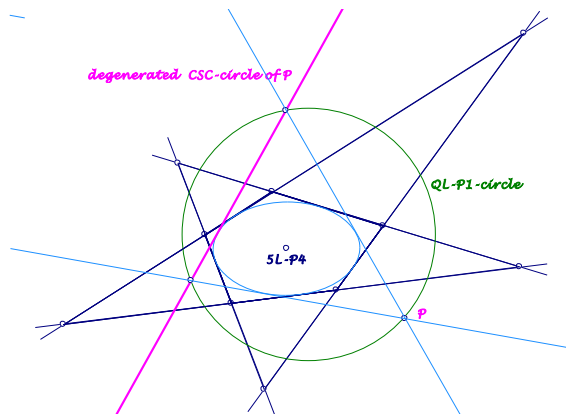
18. For a point *P* on the *QL-P1*-circle the degenerated *CSC*-circle cuts the *QL-P1*-circle in two points, whose connections with *P* are also tangents to the inscribed conic.

19. For the *QL-P1*-points the degenerated *CSC*-circles are the lines L_i of the *5-L*.

20. For intersections of a line L_i and the *QL-P1*-circle the degenerated *CSC*-circles contain the 2nd intersection and *QL-P1* of the remaining *QL*.

21. For two diametral points on the *QL-P1*-circle the degenerated *CSC*-circles intersect on the polar of *5L-Px* wrt the inscribed conic (perpendicular *5L-P1.5L-P4*).

22. The *CSC*-circles of points on the inscribed conic are tangent to the *QL-P1*-circle (for the contact points of the *5L*-lines the *CSC*-circles touch the *QL-P1*-circle in the *QL-P1* points).
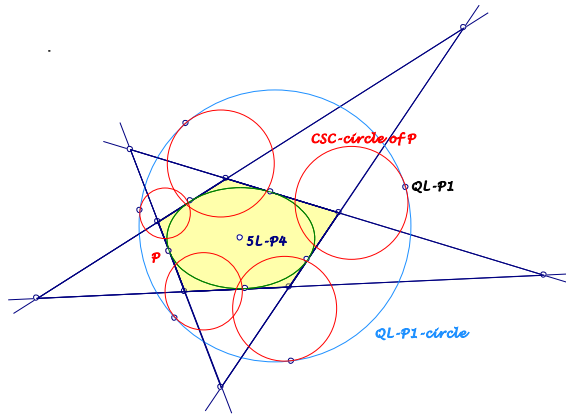
23. The *CSC*-circle of *5L-P7* contains *5L-P8* and the *CSC*-circle of *5L-P8* contains *5L-P7* (wrt the points see *QFG*-message 710).

5L-CSC-preimages:

24. Generally there are two preimages *P´* and *P´´* of a point *P*.
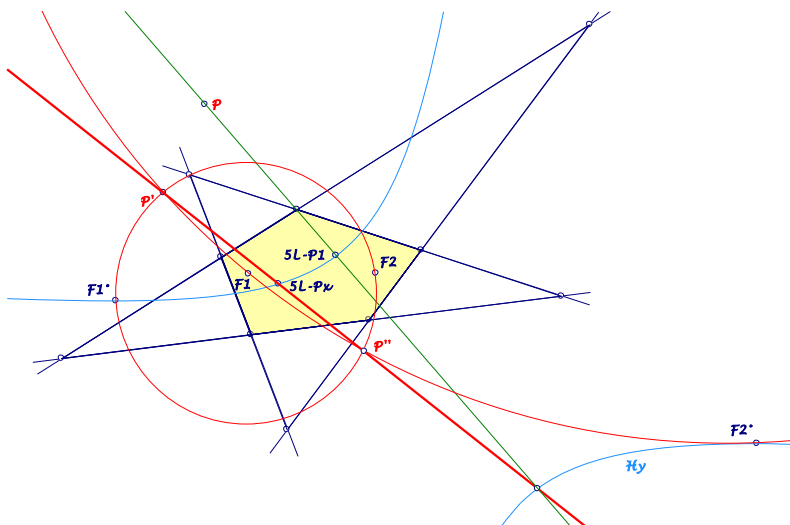
25. The two preimages *P´* and *P´´* of *P* are collinear with *5L-Px*.

26. The line $P´P´´$ is the connection of *5L-Px* and the 2nd intersection of *P.5L-P1* and the hyperbola *Hy* (see 14).

27. The two preimages $P´$ and $P´´$ of *P* are concyclic with F_1 and $F_2°$ or with $F_1°$ and F_2.

28. F_1 and $F_1°$ have the same *5L-CSC*-image F_2; F_2 and $F_2°$ the same *5L-CSC*-image F_1.

29. *5L-P1* and the inverse of *5L-Px* wrt the *QL-P1*-circle have the same *5L-CSC*-image (reflection of *5L-P1* in *5L-P4*).

30. The *CSC*-circles of $P´$ and $P´´$ are inverse wrt a circle round *P* with radius $\sqrt{PF_1 \cdot PF_2}$.

5L-CSC-fixed points

31. The *5L-CSC*-transformation has 5 fixed points (not necessary real) on the hyperbola *Hy* (see 14).
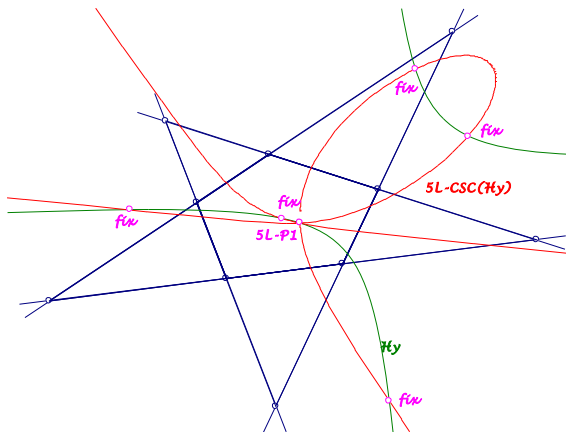
32. These fixed points are the intersections of *Hy* with its *5L-CSC*-image (without *5L-P1*).

33. Lines through a point *P* cut their *5L-CSC*-image − a conic through *5L-P1* − in two points, whose locus is a quartic *Qu(P)*, containing the fixed points of *5L-CSC* (beside *P,* the *5L-CSC*-image of *P, 5L-P1* and the intersection of *Hy* and *P.5L-Px*).

34. For points *X* on *PP´* or *PP´´* the quartics *Qu(X)* contain *P*.
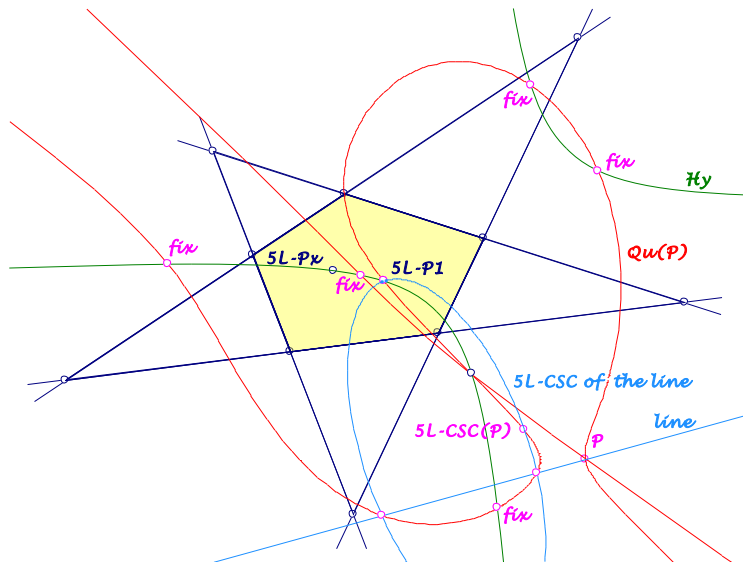
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