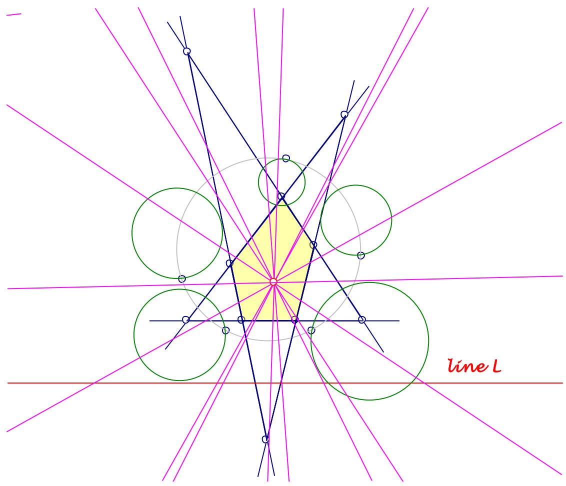
EQF-Note 2014-10-23

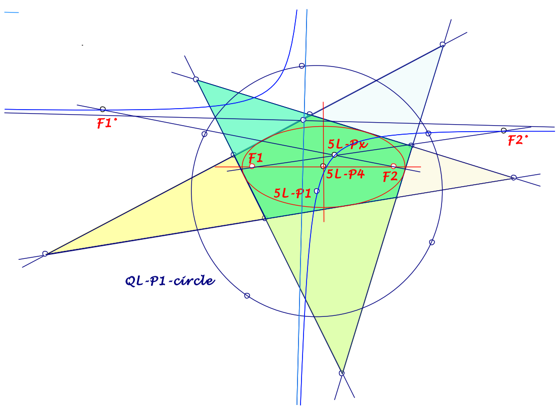
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

CSC-Line-Transformation for Pentalaterals

This CSC-related transformation maps a line to a point: For a line L the 5 CSC-circles (wrt the QL-components) have pairwise radical axes with a common point P. The geometry of this mapping is here tested with CABRI.



Some *5L*-elements will be of importance for the geometry of this transformation (see *QFG*-message 710):



… The Miquel points of the 5 *QL*-components are concyclic on the *QL-P1*-circle.
… *5L-P1* is the center of the *QL-P1*-circle.
… *5L-P4* is the center of the inscribed conic.
… F_1 and F_2 are the foci of the inscribed conic and F_1° and F_2° their inverses wrt the *QL-P1*-circle.
… *5L-Px* shall be the intersection of F_1°.F_2 and F_1.F_2°.
… *Hy* orthogonal hyperbola through *5L-P1*, *5L-Px*, *F1°* and *F2°*, centered in the midpoint of F_1°.F_2° (see *QFG* # 762).

Properties

Images of lines:

1. A line L_i of the *5L* will be mapped to the Miquel point of the remaining *QL*.

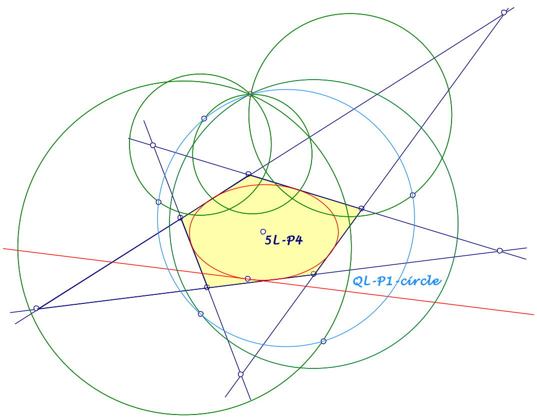
2. Lines through one focus F_i of the inscribed conic will be mapped to the other focus F_j. (In this case the image is a common point of the 5 circles.)

3. The 1st Steiner axes of the *QL*-components will be mapped to their 2nd intersection with the *QL-P1*-circle.

4. *5L-P1.5L-Px* → point at infinity of *5L-P1.5L-P4*.

5. *5L-P4.5L-Px* → inverse of *5L-Px* wrt the *QL-P1*-circle.

6. Tangents to the inscribed conic will be mapped to points on the *QL-P1*-circle. (In this case the image is a common point of the 5 circles.)

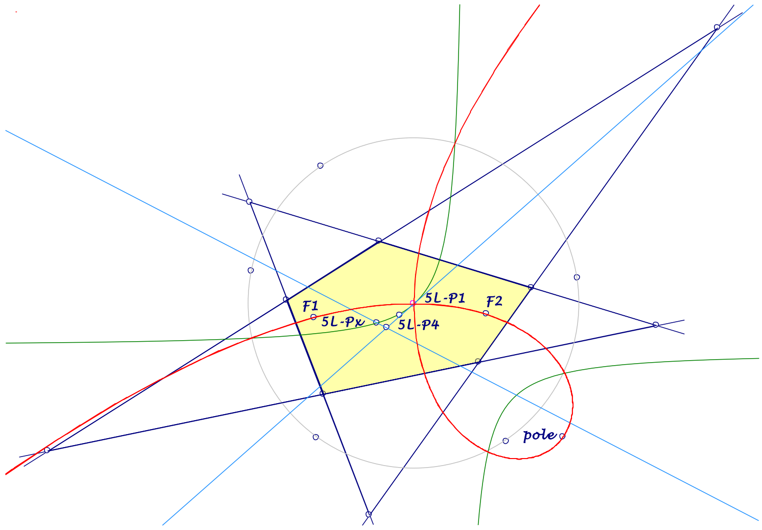


7. Tangents at the *QL-P1*-circle in the intersections with the orthogonal hyperbola *Hy* will be mapped to their contact point.

Images of line pencils

8. Lines through *5L-P4* are mapped on collinear points on the polar of *5L-Px* wrt the *QL-P1*-circle. The image of this line is *5L-P4* again.

9. Lines through *5L-Px* are mapped to points on the strophoid of *5L-P1.5L-P4* with fixed point *5L-P1* and pole in the *CSC*-line-transformation image for the connection of *5L-Px* and the reflection of *5L-P1* in *5L-P4*. This strophoid contains F_1, F_2 and is the inverse of the orthogonal hyperbola *Hy* wrt the *QL-P1*-circle.



Generalisation:

Lines are special circles. If *Ci* is a circle, then the 5 *CSC*-images wrt the *QL*-components have also a common radical center, which shall be the image of the circle.

10. The image of the *QL-P1*-circle is the reflection of *5L-Px* in *5L-P4*.

11. A circle through the focus F_i is mapped to the other focus.

12. The circumcircle wrt 3 *5L*-lines is mapped to the intersection of the remaining 2 *5L*-lines.

13. Circles round *5L-P4* have collinear images on *5L-P1.5L-Px*.

14. Circles round *5L-P1* have images on an orthogonal hyperbola through *5L-P1*, *5L-Px*, F_1, F_2 (centered in *5L-P4*, asymptotes parallel to those of *Hy*).

15. The inverse of this orthogonal hyperbola (wrt the *QL-P1*-circle) is the strophoid of a line, connecting *5L-P1* and the center of *Hy*, with fixed point *5L-P1* and pole in the inverse (wrt *QL-P1*-circle) of the reflection of *5L-P1* in *5L-P4*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de