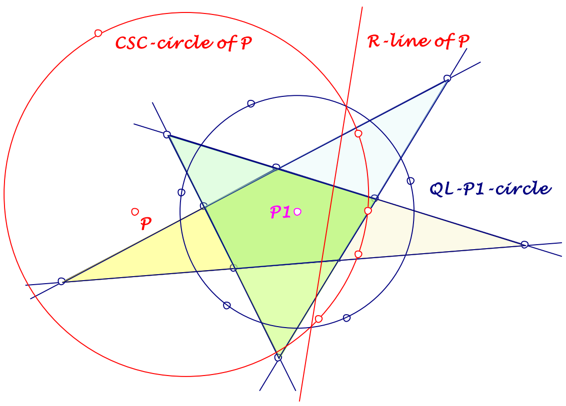
EQF-Note 2014-11-06

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

CSC-Circles for Pentalaterals

The transformation QL-Tf1 – shortened CSC – is the central transformation for quadrilaterals. For a pentalateral with 5 QL-components there are 5 concyclic CSC-images of a point. The geometry of these CSC-circles will be Cabri-researched, considering points on lines and circles wrt a pentalateral.



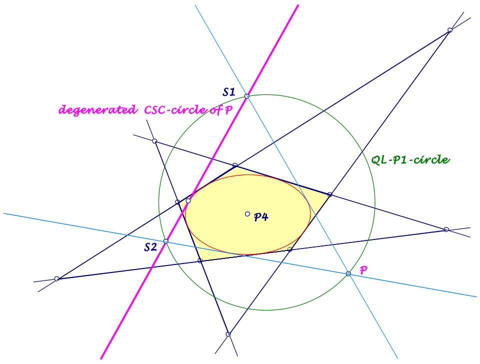
Preliminary remarks

Some *5L*-elements will be of importance for the geometry of *CSC*-circles (see *QFG*-message 710):
… The Miquel points of the 5 *QL*-components of a *5L* are concyclic on the *QL-P1*-circle.
… *5L-P1* = P_1 is the center of the *QL-P1*-circle.
… *5L-P4* = P_4 is the center of the inscribed conic.
… F_1 and F_2 are the foci of the inscribed conic and $F_1°$ and $F_2°$ their inverses wrt the *QL-P1*-circle.
… *5L-Px* = P_x shall be the intersection of $F_1°.F_2$ and $F_1.F_2°$.
… *Hy* is the orthogonal hyperbola through P_1, P_x, $F_1°$, $F_2°$ with center *Z*.

For basic properties of *CSC*-circles see *EQF*-Note 2014-10-21 in *QFG*-message 769. Here we need the special property:

- **The *CSC*-circle for a point on the *QL-P1*-circle degenerates to a line, tangent to the inscribed conic.**

This line can be easily constructed: Consider the tangents from P (on the QL-$P1$-circle) to the inscribed conic and their 2nd intersections S_1 and S_2 with the QL-$P1$-circle, then S_1S_2 is the degenerated CSC-circle of P.



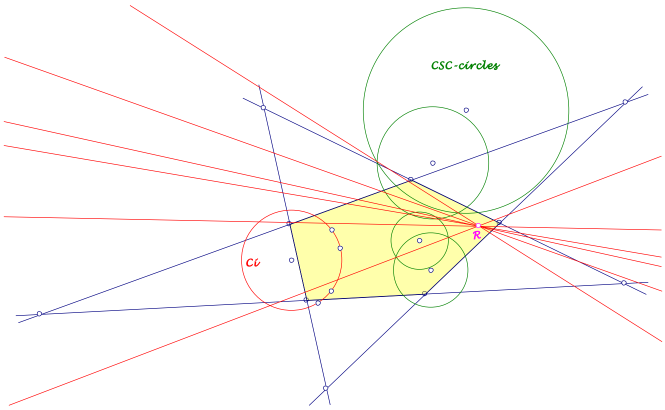
The *R*-line of a point

Definition: The *R*-line of a point *P* is the radical axis of the *CSC*-circle of *P* and the *QL-P1*-circle.

- **The *R*-line for an intersection of two *5L*-lines is the line between the corresponding *QL-P1*-points.**
- **The *R*-line for the *QL-P1*-point of a *QL*-component is the remaining *5L*-line.**
- **The *R*-line of a point *P* on the *QL-P1*-circle is the degenerated *CSC*-circle of *P*.**
- **The *R*-line of a point *P* on the inscribed conic is tangent to the *QL-P1*-circle.**
- **The *R*-line of a contact-point *P* of the inscribed conic is the tangent to the *QL-P1*-circle in the corresponding *QL-P1*-point.**
- **The *R*-line for a focus F_i of the inscribed conic is the perpendicular bisector of the other focus F_j and its reflection in the *QL-P1*-circle.**
- **The *R*-line of P_1 is the polar of P_x wrt the inscribed conic.**
- **The *R*-line of P_4 is a perpendicular line to P_1P_x through the inverse of P_x wrt the *QL-P1*-circle.**

The *R*-point of a line or circle

Definition: The *CSC*-circles of points on a given line or circle have radical axes with a common point (*R*-point of a line or circle).

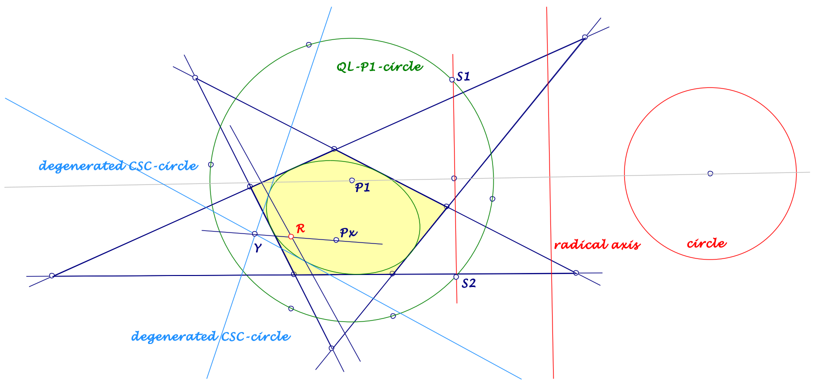


Construction of the *R*-point for a circle:

If there are two intersections of the circle and the *QL-P1*-circle, the corresponding degenerated *CSC*-circles give the *R*-point.

If there are not two intersections of the circle and the *QL-P1*-circle, consider the radical axis and a parallel through its pole wrt the *QL-P1*-circle, which cuts the *QL-P1*-circle in S_1 and S_2, whose degenerated *CSC*-circles intersect in *Y*. The intersection of YP_x and the polar of *Y* wrt the inscribed conic give the *R*-point of the circle.

The R-point of a line can be constructed in the same way, using the line as radical axis.



- **The *R*-line of a point and the *R*-point of a line are inverse transformations.**

- **The *R*-point of the *CSC*-circle of *P* is *P* again.**

- **All circles with the same radical axis wrt the *QL-P1*-circle have the same *R*-point.**

- **The *R*-point of a *5L*-line is the *QL-P1*-point for the remaining *QL*-component.**

- **The *R*-point of a tangent in *P* at the *QL-P1*-circle is the contact-point of the degenerated *CSC*-circle of *P* and the inscribed conic.**

- **The *R*-point of a tangent in *P* at the inscribed conic is the contact-point of the *CSC*-circle of *P* and the *QL-P1*-circle.**

- **The *R*-point of the line *L = 5L-P1.5L-P2.5L-P3* is the pole (wrt the inscribed conic) of the line through P_x and the 2nd intersection of *L* and the hyperbola *Hy*.**

Further example: Consider the circle of the 5 *QL-P4*-points (see *QFG*-message 710): The *R*-point of this circle has a *CSC*-circle, centered on the line 5L-P1.5L-P2.5L-P3.

***R*-points for *Hy*-tangents**

- **The *R*-points for tangents at the orthogonal hyperbola *Hy* (through P_1, P_x, $F_1°$, $F_2°$, centered in *Z*) give a parabola: directrix P_1P_x, focus is the *R*-point of a perpendicular line to P_1Z in *Z*.**

***R*-points of concentric circles**

- **The *R*-points for concentric circles with center *P* give a line through P_x and the pole (wrt the inscribed conic) of the *R*-line of *P*.**

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