EQF-Note 2014-12-16

Hodgson's Directed n-Lines

These are first remarks wrt an article of J. E. Hodgson: "Orthocentric Properties of the Plane Directed n-Line"(1912).
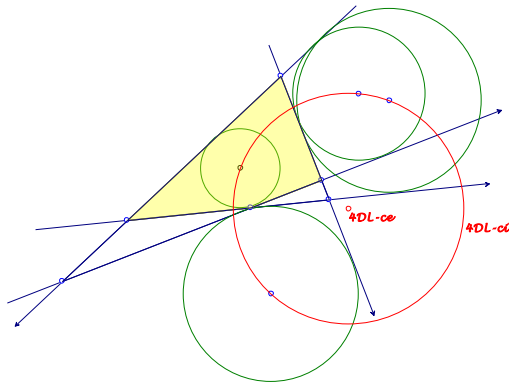

(1) If the lines of a 3-line have a direction, there is a unique point (p.200) equidistant to the 3 lines as center of a circle contacting the lines. This is the first step for a chain of centers and center circles of directed n-lines (shortened (*nDL-ce* and *nDL-ci*).

(2) For a directed 4-line there are four *3DL-ce* on the center circle *4DL-ci* (p.203). This circle is one of the 8 Steiner circles of a quadrilateral.
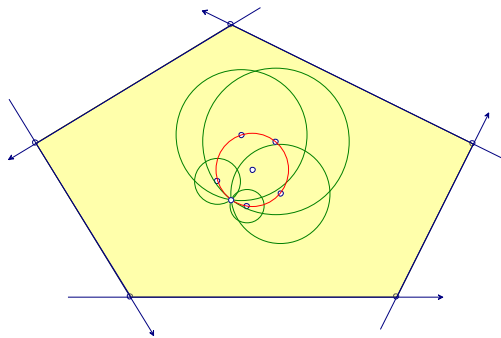
(3) For a directed 5-line there are five *4DL-ce* on the center circle *5DL-ci* (p.227). Not mentioned by Hodgson: The five center circles *4DL-ci* have a common point *5DL-px* on the center circle *5DL-ci*.
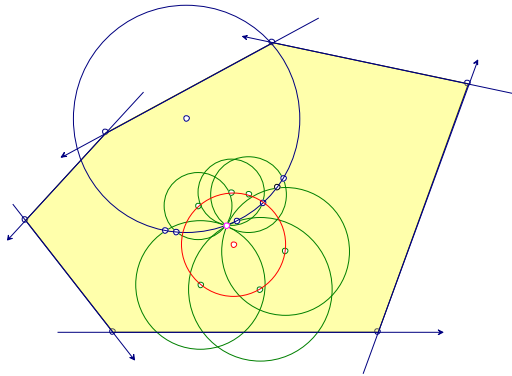
(4) For a directed 6-line there are six *5DL-ce* on the center circle *6DL-ci*. Not mentioned by Hodgson: The six center circles *5DL-ci* have a common point *6DL-px*. This common point is concyclic with the six points *5DL-px* of the directed 5-lines on a circle *6DL-cix*.
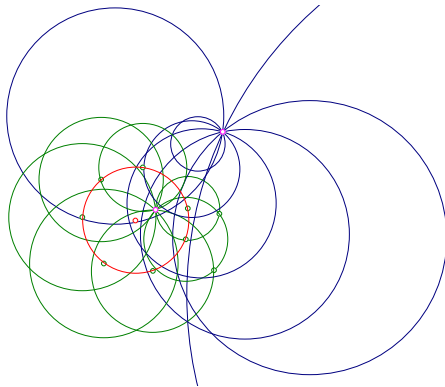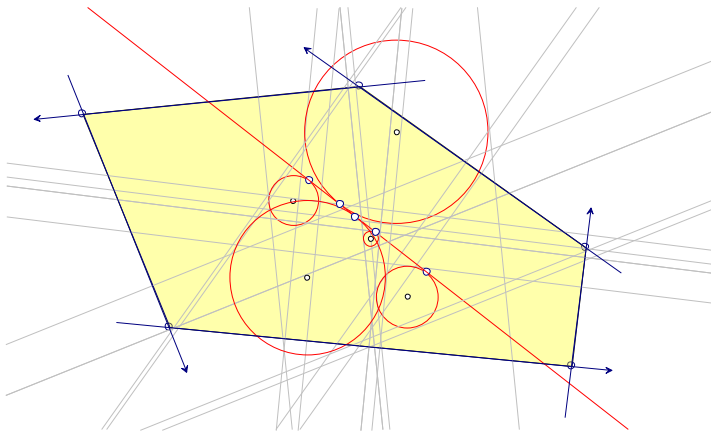

(5) For a directed 7-line (not in the figure) there are seven *6DL-ce* on the center circle *7DL-ci* (red). Not mentioned by Hodgson: The seven center circles *6DL-ci* (green) have a common point as well as the seven circles *6DL-cix* (blue).

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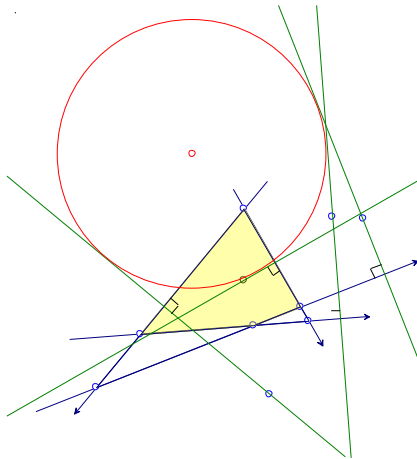
(6) For directed 4-lines Hodgson mentioned a second circle (p.204) inscribed the perpendiculars of the *3DL-ce* wrt the line left out.

(7) For a directed 5-line the five second circles of the directed 4-lines have a common tangent (p.207).

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(8) A n-line has 2^{n-1} directed n-lines (reversal of all directions shall give the same directed n-line). So a 4-line has 8 directed 4-lines and a 5-line 16 directed 5-lines.

(9) For a 4-line the 8 center circles *4DL-ci* are the Steiner circles with centers on two perpendicular lines (1st and 2nd Steiner axis).

Final questions:

For a 5-line Hodgson proved, that the centers of the center circles of the 16 directed 4-lines are on a rectangular hyperbola (p.230). This can be verified by construction for 5 tangents of a parabola (p.228) but not in general.

For a 6-line Hodgson proved, that the centers of the center circles of the 32 directed 5-lines are on two perpendicular lines (p.231). This cannot be verified by construction.

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