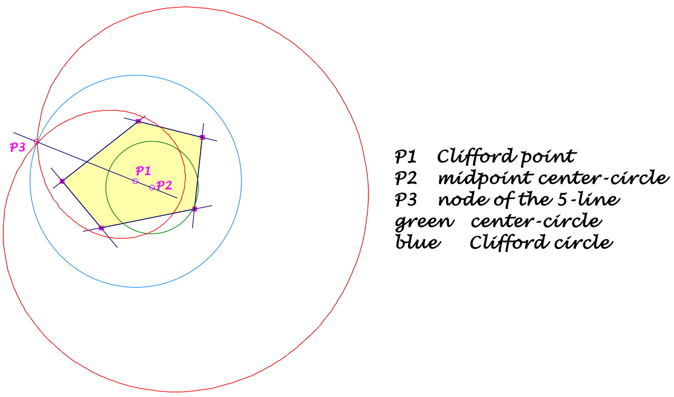
EQF-Note 2015-01-02

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Morley's Limaçon as Inverse of a Conic

Morley described in "On the metric geometry of the plane n-line" a limaçon for n-lines. Here are gathered some properties of this curve for a 5-line. Finally result is a conic for n-lines inverse to the limaçon.



Notations

Clifford chain (§4): There is a recursive definition of Clifford points for an even number of lines and of Clifford circles for an odd number of lines, beginning with the circumcircle for 3-lines.
Clifford point of *2n*-lines is the common point of the Clifford circles for *2n-1* of *2n* lines (Miquel point *QL-P1* for 4-lines).
Clifford circle of *(2n+1)*-lines is the circumcircle of the Clifford points for *2n* of *2n+1* lines.

Center-circles (§2): A recursive definition begins with the circumcircle for 3-lines.
The center-circle of *n*-lines is the circumcircle of the midpoints of the center-circles for *n-1* of *n* lines (Miquel circle *QL-Ci3* for 4 lines).

Node of a *n*-line (§3)**:** Common point of the center-circles for *n-1* of *n* lines (Miquel point *QL-P1* for 4-lines).

Morley's limaçon for *n*-lines is the envelope of circles through the node, centered on the center-circle.

These circles are Morley's "penosculants" (p.102), which he studies in §5.

"Limaçon" of a 4-line

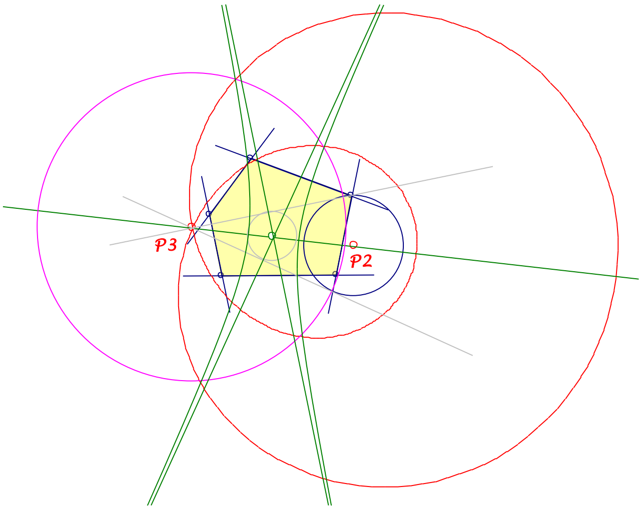
For a 4-line the node is the Miquel point *QL-P1* on the center-circle *QL-Ci3*. So the limaçon degenerates to a cardioïd.
Circles round points on *QL-Ci3* through *QL-P1* envelop the cardioïd *QL-Qu1*.
This cardioïd can also be considered …
… as locus of the reflections of *QL-P1* in tangents at *QL-Ci3*.
… as *CSC*-image of the inscribed parabola *QL-Co1* (see *EQF*).
… as pedal curve: Consider a circle through *QL-P1* and centered in the reflection of *QL-P1* in *QL-P4*. The pedal points of *QL-P1* wrt tangents at this circle give the cardioïd.
… as catacaustic of a circle round *QL-P4* through a point, which divides *QL-P1.QL-P4* with ratio *-4/3*. Rays from this point envelop with their reflections at the circle the cardioïd.

Limaçon of a 5-line

We shall use the following notation wrt a 5-line (see *QFG* message 710):

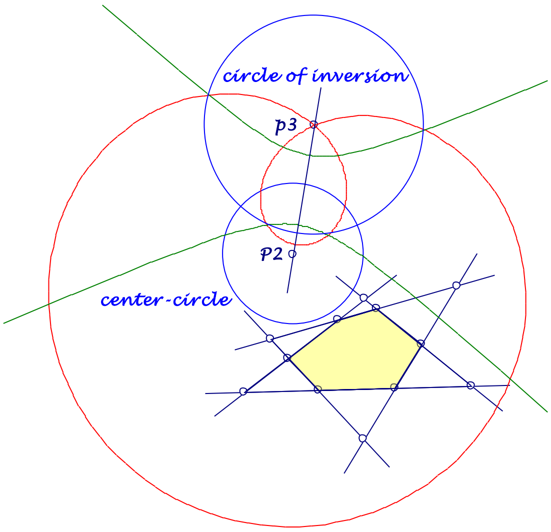
P_1 midpoint Clifford circle,
P_2 midpoint center-circle,
P_3 node of the 5-line.

- The node P_3 is the inverse of P_1 wrt the center-circle.
- The limaçon is the envelope of circles through the node P_3, centered on the center-circle (see above).
- The limaçon is the envelope of circles, centered on a circle round P_2 through P_3, which orthogonally intersect the Clifford circle.
- The limaçon is the locus of the reflections of P_3 in tangents at the center-circle.
- The limaçon is invariant under an inversion wrt the Clifford circle.
- The vertices of the limaçon are the reflections of the node P_3 in the intersections of P_1P_2 and the center-circle.
- The limaçon is the pedal curve of P_3 wrt the tangents of the Thales circle about the vertices.
- The limaçon is the inverse of a special conic: Foci of the conic are P_2 and P_3, center the midpoint. The circle about the vertices of the conic contacts the tangents from P_3 to the center-circle. Circle for inversion is a circle round P_3, orthogonal intersecting the center-circle.



Generalisation

Morley’s limaçon of a n-line depends only on the center-circle (with midpoint $nL\text{-}P_2$) and the common point $nL\text{-}P_3$ of the center-circles of $n\text{-}1$ of n lines.
Morley’s limaçon is the inverse of a conic ($n>4$):
… The circle for inversion is centered in $nL\text{-}P_3$ orthogonal intersecting the center-circle.
… The conic is the envelope of the perpendicular bisectors of $nL\text{-}P_3$ and points of the center circle.
Or let X be points on the center circle:
… The conic is the locus for intersections of perpendicular bisectors of $X.nL\text{-}P_3$ and $X.nL\text{-}P_2$.



Final question:
What is the relation of the conic to the reference n-line?

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de