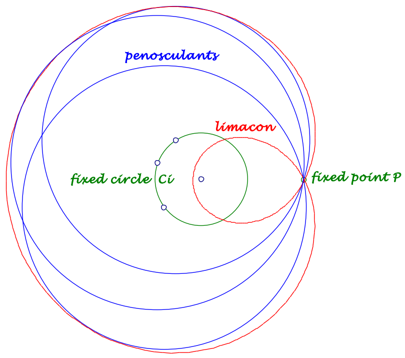
EQF-Note 2015-01-03

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

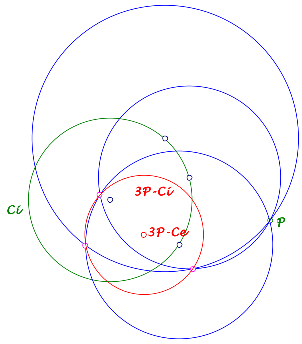
Morley's Penosculants of the Limaçon

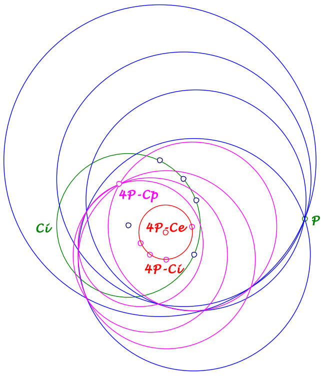
Morley describes in "On the metric geometry of the plane n-line" the geometry of circles centered on a fixed circle through a fixed point – named "penosculants of the limaçon". The geometry of n penosculants corresponds to the geometry of n-lines. Here are gathered some figures in addition to Morley's calculations.



Penosculants of the Limaçon

In §5 Morley describes penosculants as circles through a point *P* and centered on a circle *Ci*, their envelope is a limaçon.
… If we consider three penosculants, there are three 2nd intersections with circumcircle *3P-Ci* and center *3P-Ce*.
… For four penosculants there are four *3P-Ci* with common point *4P-Cp* and centers on a circle *4P-Ci* with center *4P-Ce*.



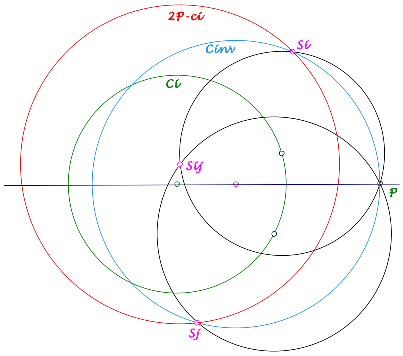


For five penosculants there are five *4P-Ci* with common point *5P-Cp* and centers on a circle *5P-Ci* with center *5P-Ce*…
This gives a chain analog to Morley's center-circle theorem for n-lines (p.107).

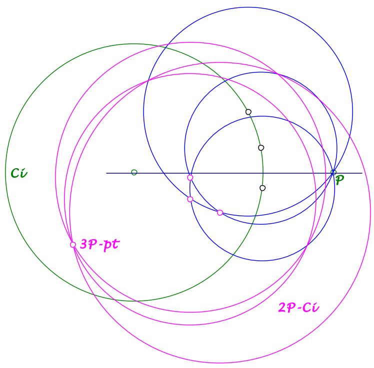
If we take for *Ci* the center-circle of a n-line and as penosculants the n center-circles of the *(n-1)*-lines and as *P* their common point, then there will be further points and circles for n-lines.

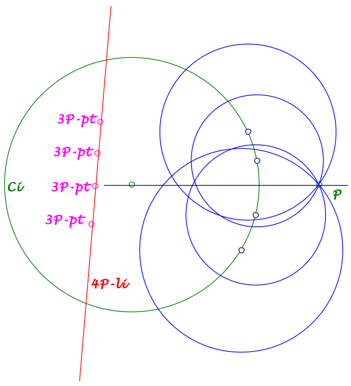
Inversion of the Limaçon

The limaçon is invariant under an inversion. The circle of inversion *Cinv* contains the node *P* and has its center in the inverse of *P* wrt the fixed circle *Ci*.
… If we consider two penosculants p_i and p_j, there is a 2nd intersection $S_{i,j}$ and 2nd intersections S_i and S_j with the circle of inversion *Cinv*. The circle round $S_{i,j}$ through S_i and S_j shall be named *2P-ci*.



… If we consider three penosculants, there are three circles *2P-ci* with a common point *3P-pt*.
… For four penosculants there are four points *3P-pt* on a line *4P-li*.

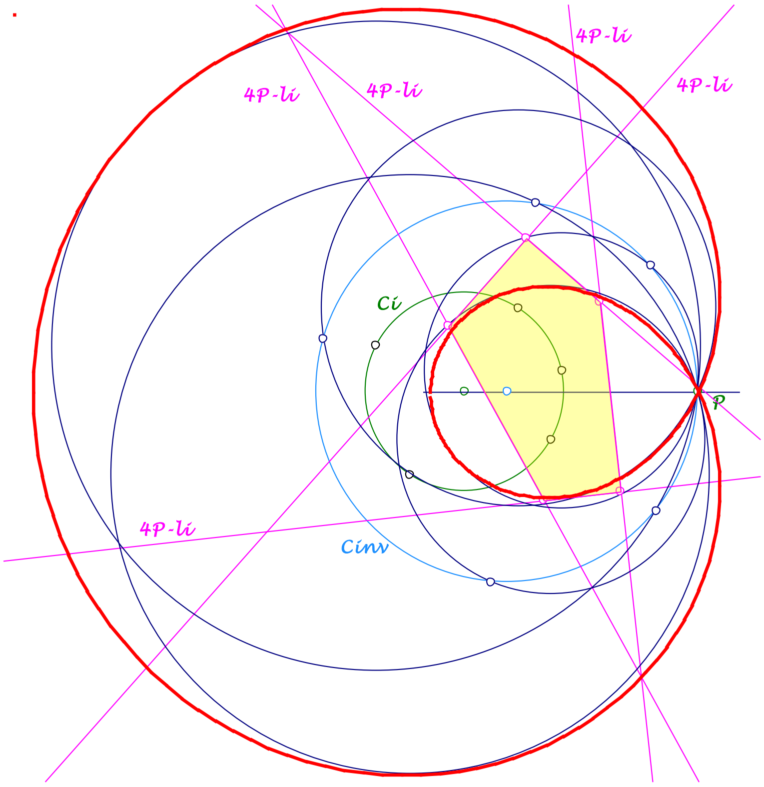




… For five penosculants there are five lines *4P-li*, defining a 5-line with the following properties:

- The five Miquel Circles *QL-Ci3* of this 5-line are the five penosculants with common point *P*.
- The five Miquel Circumcenter *QL-P4* for this 5-line are the midpoints of the penosculants.
- The five Miquel Points *QL-P1* for this 5-line lie on the circle of inversion *Cinv* in the 2nd intersections with the penosculants.

- Morley’s center-circle of this 5-line is the fixed circle *Ci* of the penosculants.



These last properties are not explicit mentioned in Morley’s paper, but they will be the background of his final remark (p.106):
“Naturally then five penosculants will give five lines and thus we get the MIQUEL figure as completed by KANTOR.”

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