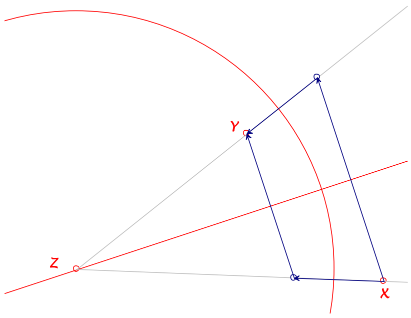
EQF-Note 2015-01-08

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

CSC-Transformation for 2n-Lines

The CSC-transformation QL-Tf1 is an interesting mapping for 4-lines (see EQF). Here is a generalization for 2n-lines, orientated at an involution, which Morley mentioned in "On the metric geometry of the plane n-line" for 2n lines tangent to a conic. With an inversion we can assign to the reference 2n-line a 2nd 2n-line with an inscribed conic. This inversion, followed by Morley's involution for the 2nd 2n-line and then the inversion again, this gives the 2n-CSC-transformation for a 2n-line.



Preliminary Remarks

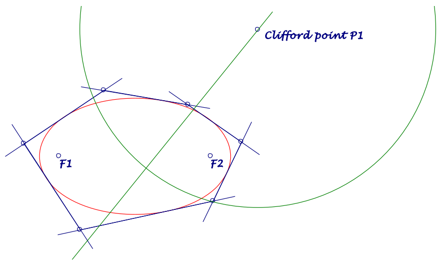
The *CSC*-transformation is an involution, determined by a center *Z* and a pair *X*, *Y* of *CSC*-partners. This involution consists of a reflection in the angle bisector of $\angle XZY$ and an inversion wrt a circle round *Z* with radius $\sqrt{ZX \cdot ZY}$.
For a 4-line *Z* is the Miquel point *QL-P1* and *X*, *Y* can be two opposite intersections of the 4-line.

Morley's results

In §6 Morley researched 2n-lines with an inscribed conic. Here are cited his results:

"This involution has the following properties:
- (i) Its center is the Clifford point of the 2p-lines.
- (ii) The foci are a pair of the involution I_1^2.
- (iii) The Clifford point of 2q lines and that of the remaining 2(p-q) lines are a pair of I_1^2. The Clifford point of two lines means merely their intersection.

(iv) The Clifford circle of 2q-1 lines and that of the remaining lines are partners. The Clifford circle of a line is merely the line itself.”



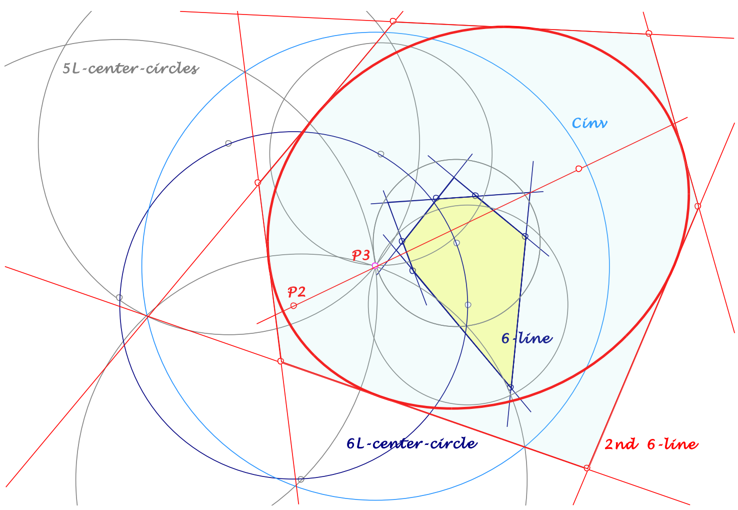
For a 4-line, which has always an inscribed conic, this involution is the *CSC*-transformation.

For 6-lines with an inscribed conic this involution is the transformation mentioned in *QFG* #784, #861, #864.

Generalization

For a 2n-line we shall use the following points (generalized notation of *QFG* #710) …
… P_1 Clifford point of the 2n-line,
… P_2 center of the 2n-center-circle,
… P_3 common point of the (2n-1)-center-circles of the 2n-line.

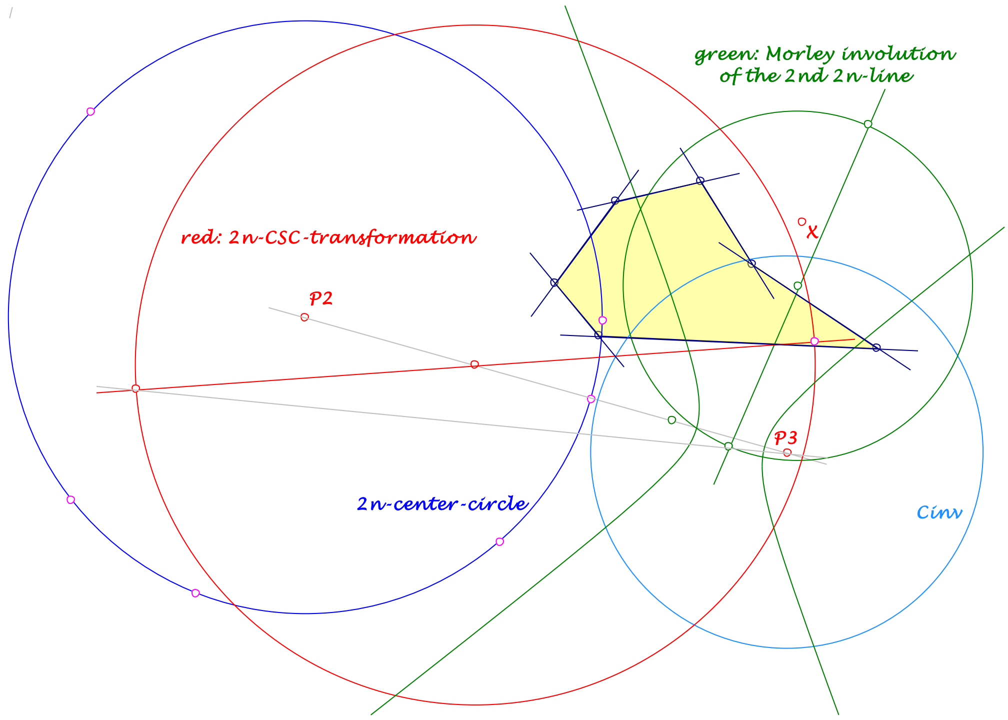
We consider a 2n-line and the 2n center-circles for 2n-1 of 2n lines.
… These (2n-1)-center-circles intersect in the point P_3 and their centers give the 2n-center-circle with center P_2.
… In addition we use an inversion wrt an arbitrary circle C_{inv} round *P3*.
… The images of the (2n-1)-center-circles are 2n lines tangent to a conic, defining a 2nd 2n-line.
… This conic has one focus P_3 (a special example in *QFG* # 918). For the 2nd focus take the inverse of P_3 wrt the 2n-center-circle and then the C_{inv}-inverse of this point.



… **The 2n-CSC-transformation is the following chain of mappings:**

Inversion wrt a circle C_{inv} round P_3,
Morley's involution wrt the 2nd 2n-line,
Inversion wrt the circle C_{inv} again.

… The 2n-*CSC*-transformation for a 4-line is the *CSC*-transformation *QL-Tf1*.
… The center of the 2n-*CSC*-transformation is the inverse of P_3 wrt the 2n-center-circle.
… A pair of 2n-*CSC*-partners are P_3 and the C_{inv}-inverse X of the Clifford point of the 2nd 2n-line.
… The fixed points of the 2n-*CSC*-transformation are the C_{inv}-inverses of the fixed points of the Morley involution for the 2nd 2n-line.



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