EQF-Note 2015-01-18

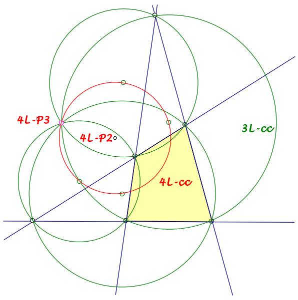
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Center-Circles and Clifford's Chain for n Lines and n Penosculants

In his paper "On the metric geometry of the plane n-line" Morley treated center-circles and Clifford's chain for n lines as well as for n penosculants. Penosculants are circles through a fixed point, centered on a fixed circle. Center-circles for n-1 of n lines are the penosculants of a n-line. Here is tried to describe relationships between points and circles for n lines and the corresponding penosculants.

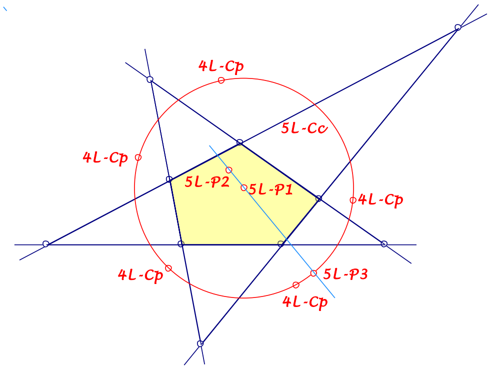
Center-circles of n-lines (§2):

A recursive definition begins with the circumcircle for 3-lines. The center-circle *nL-cc* of *n*-lines is the circumcircle of the midpoints of the center-circles for *n-1* of *n* lines. The midpoint of the center-circle *nL-cc* shall be *nL-P2*. The center-circles for *n-1* of n lines have a common point *nL-P3* (nomination as in *QFG #710* for 5-lines).



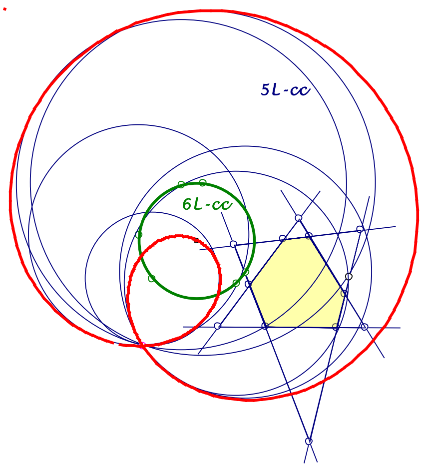
Clifford's chain (§4)

There is a recursive definition of Clifford points 2*nL-Cp* for an even number of lines and of Clifford circles *(2n+1)L-Cc* for an odd number of lines, beginning with the intersection of 2 lines. Clifford circle of *(2n+1)*-lines is the circumcircle of the Clifford points for *2n* of *2n+1* lines. Let *(2n+1)L-P1* be the midpoint of the Clifford circle. Clifford point of *2n*-lines is the common point of the Clifford circles for *2n-1* of *2n* lines.
Special: For a 3-line the circumcircle is the Clifford circle *3L-Cc* as well as the center-circle *3L-cc*. For a 4-line the Clifford point *4L-Cp* = *4L-P3* is the Miquel point. For a 5-line the points *5L-P1, 5L-P2, 5L-P3* are collinear with *5L-P3* on the Clifford circle *5L-Cc* and *5L-P1* the inverse of *5L-P3* wrt the center-circle *5L-cc*.



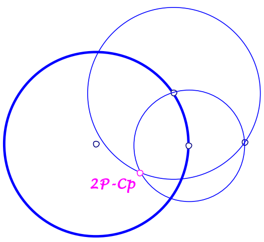
Penosculants (§3)

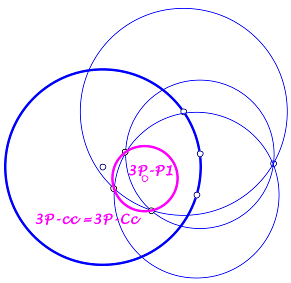
Morley defines penosculants as circles through a fixed point centered on a fixed circle. For example the center-circles for *n-1* of *n* lines give *n* penosculants. It is evident, that penosculants define a limaçon.



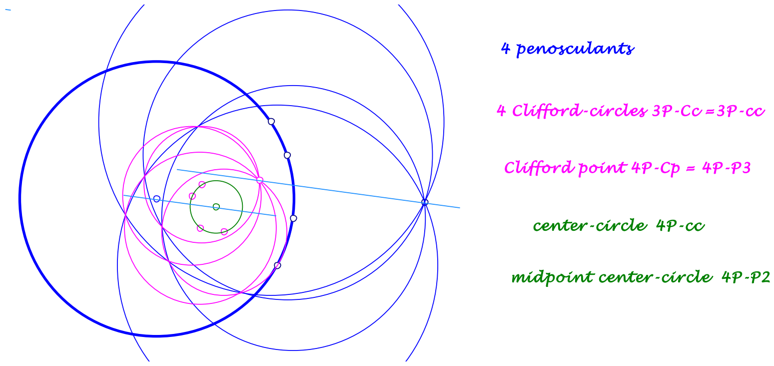
Center-circles and Clifford's chain for n penosculants (§5)

There is an analog recursive definition of center-circles *nP-cc* and of Clifford points *2nP-Cp* for an even number and of Clifford circles *(2n+1)P-Cc* for an odd number of penosculants, beginning with the 2nd intersection of 2 penosculants for the Clifford chain and with the circumcircle of the 2nd intersections of 3 penosculants for the center-circles. Analog we can consider points *nP-P1* (for odd *n*), *nP-P2* and *nP-P3*.

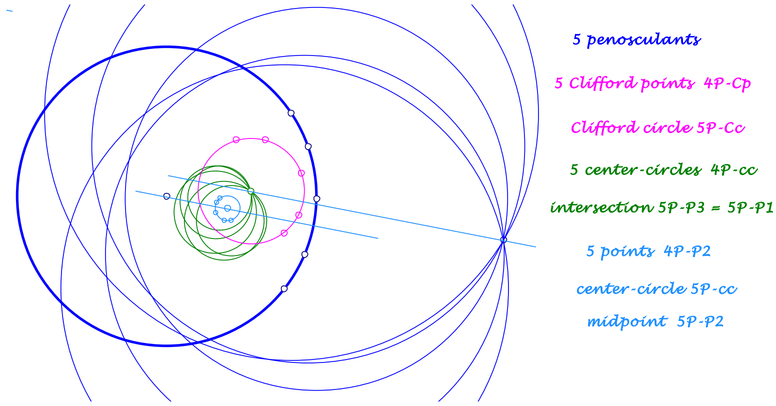




For 4 penosculants the four circles *3P-Cc* = *3P-cc* have the Miquel point as common point *4P-Cp* = *4P-P3* and midpoints on the center-circle *4P-cc*. This gives a new constellation for 4 penosculants, whose center-circles reproduce the reference fixed point and circle. Let *P2* be the center of the fixed circle and *P3* the fixed point, then *P2.4P-P2* is parallel *P3.4P-P3*.

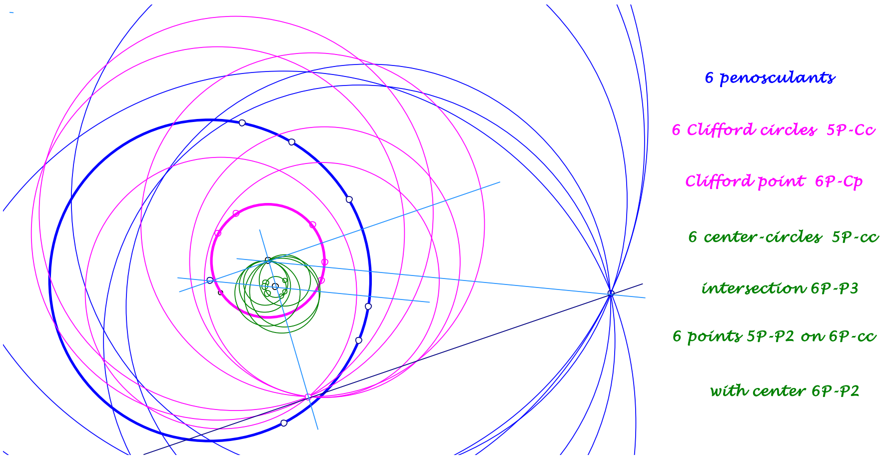


For 5 penosculants the five Clifford points *4P-Cp* give the Clifford circle *5P-Cc*. The five center-circles *4P-cc* intersect in the point *5P-P3*, which is the midpoint of the Clifford circle. The midpoints of the center-circles *4P-cc* give the center-circle *5P-cc* with midpoint *5P-P2*. Let *P2* be the center of the fixed circle and *P3* the fixed point, then *P2.5P-P2* is parallel *P3.5P-P3*.



For 6 penosculants there is something new: The six Clifford circles *5P-Cc* are centered on a circle with midpoint *6P-P3*. This is not valid for the Clifford circles *6L-Cc* of 6 lines.
Let *P2* be the center of the fixed circle and *P3* the fixed point, then *P2.6P-P2* is parallel *P3.6P-P3*. This seems to be valid in general.
Further holds: The points *6P-P2*, *6P-P3*, *6P-Cp* are collinear and the lines *P2.6P-P3*, *P3.6P-Cp* are parallel.

So far examples for Morley's result in §5, that the center-circle theorem and Clifford's chain also exist for penosculants.

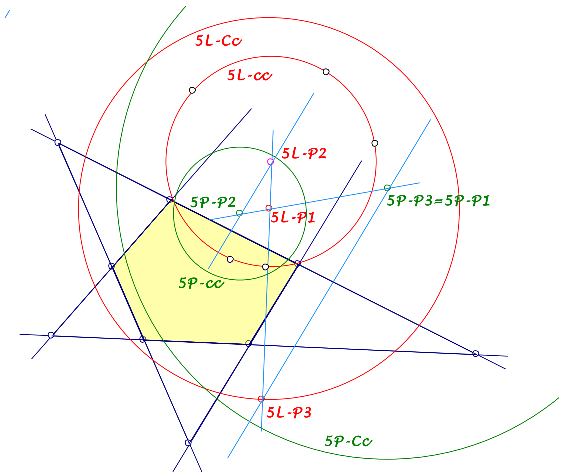


n-lines and their penosculants

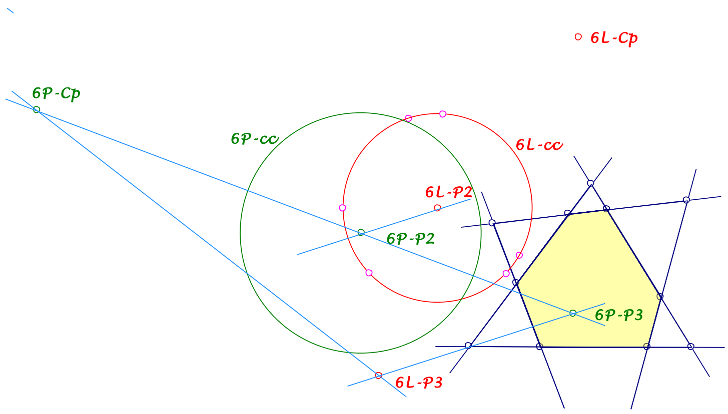
The *n* center-circles *(n-1)L-cc* of *n* lines are the penosculants of a *n*-line: *nL-P3* is the fixed point *P3*, the center-circle *nL-cc* is the fixed circle with center *nL-P2* = *P2*. We can have a look on the following points and circles:

center-circles:	*nL-cc and nP-cc*,
Clifford circles (*n* odd):	*nL-Cc and nP-Cc*,
midpoint Clifford circles (*n* odd):	*nL-P1 and nP-P1*,
Clifford points (*n* even):	*nL-Cp and nP-Cp*,
midpoint center circles:	*nL-P2 and nP-P2*,
intersection of *(n-1)*-center-circles:	*nL-P3 and nP-P3*.

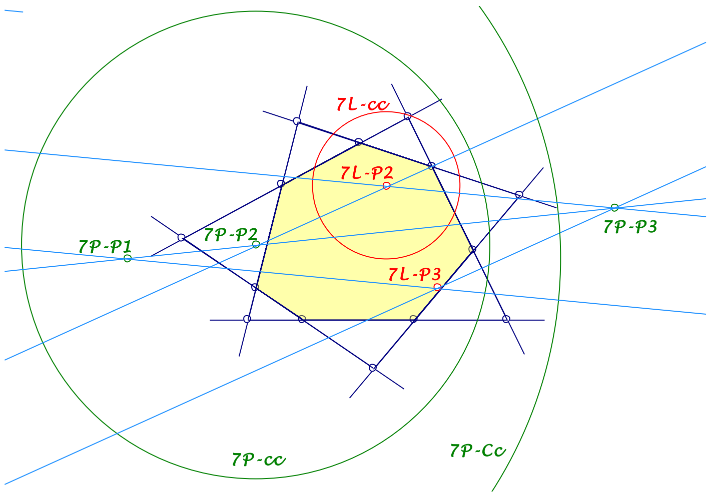
4-line:
4L-cc = *QL-Ci3*, *4L-Cp* = *4L-P3* = *QL-P1*, *4L-P2* = *QL-P4*, *4P-cc* is the line at infinity, so *4P-Cp*, *4P-P2*, *4P-P3* are not defined.



5-Line
5L-P1, *5L-P2*, *5L-P3* are the collinear points as defined in *QFG #710*. *5L-cc* is the *QL-P4*-circle, *5L-Cc* is the *QL-P1*-circle. Already mentioned: *5L-P2.5P-P2* is parallel *5L-P3.5P-P3* (see *QFG #931*). New is the circle *5P-Cc*, centered in *5P-P1* = *5P-P3* collinear with *5L-P1* and *5P-P2*.



6-line
6L-P2.6P-P2 is parallel *6L-P3.6P-P3* (see *QFG #931*).
6L-P2.6P-P3 is parallel *6L-P3.6P-Cp*.
6P-P2, *6P-P3* and *6P-Cp* are collinear.



7-line
7L-P2.7P-P2 is parallel *7L-P3.7P-P3* (see *QFG #931*).
7L-P2.7P-P3 is parallel *7L-P3.7P-P1*.
7P-P1, *7P-P2* and *7P-P3* are collinear.

Final guess
There are limits of CABRI observations, but perhaps holds:

… ***nL-P2.nP-P2*** **parallel** ***nL-P3.nP-P3***
… (*n* even) ***nL-P2.nP-P3*** **parallel** ***nL-P3.nP-Cp***
… (*n* nod) ***nL-P2.nP-P3*** **parallel** ***nL-P3.nP-P1*.**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de