EQF-Note 2015-01-25

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Basic Elements of 6-Line Geometry

Background for this excursion in 6L-geometry is Morley's paper "On the geometry of the plane n-line" wrt center-circles, Clifford's chain and penosculants of the limaçon. There remains the guess, that these results can be generalized for 2n-lines.

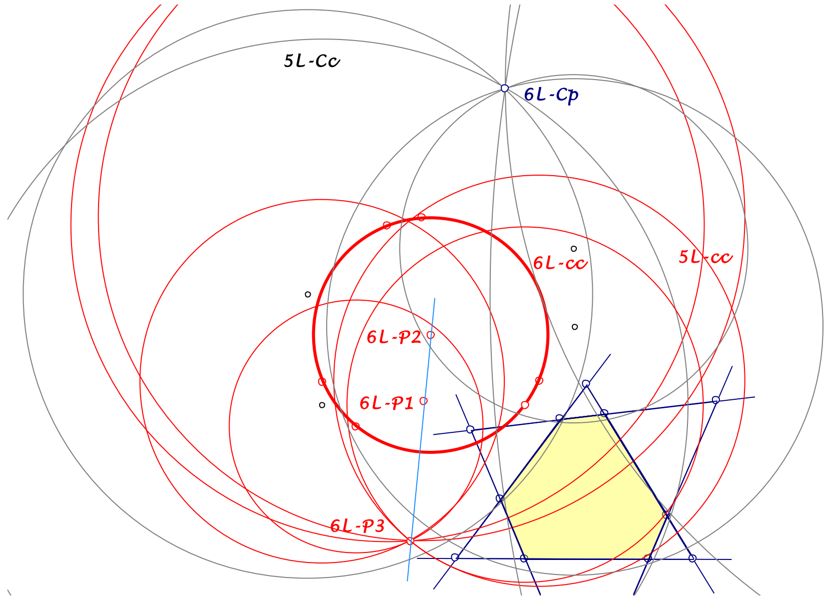
Nominations

We shall have a look on the following elements, which are explained in *QFG* message *937* and *922*:

center-circles:	*nL-cc* and *nP-cc*,
midpoint center circles:	*nL-P2* and *nP-P2*,
intersection of *(n-1)*-center-circles:	*nL-P3* and *nP-P3*,
inverse of *nL-P3/nP-P3* wrt *nL-cc/nP-cc*:	*nL-P1* and *nP-P1*
Clifford circles (*n* odd):	*nL-Cc* and *nP-Cc*,
Clifford points (*n* even):	*nL-Cp* and *nP-Cp*.

The *n* penosculants of an *n*-line are the center-circles of *n-1* of *n* lines. Their midpoints lie on the center-circle of the *n*-line and their common point is *nL-P3*. Morley considers center-circles and Clifford points / circles also for penosculants.

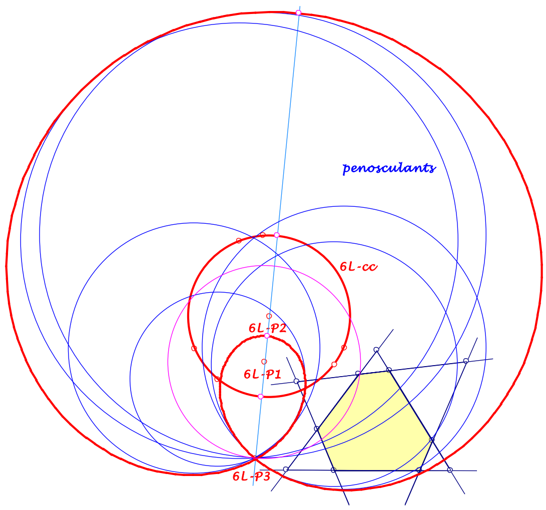
6L-cc, 6L-P1, 6L-P2, 6L-P3, 6L-Cp



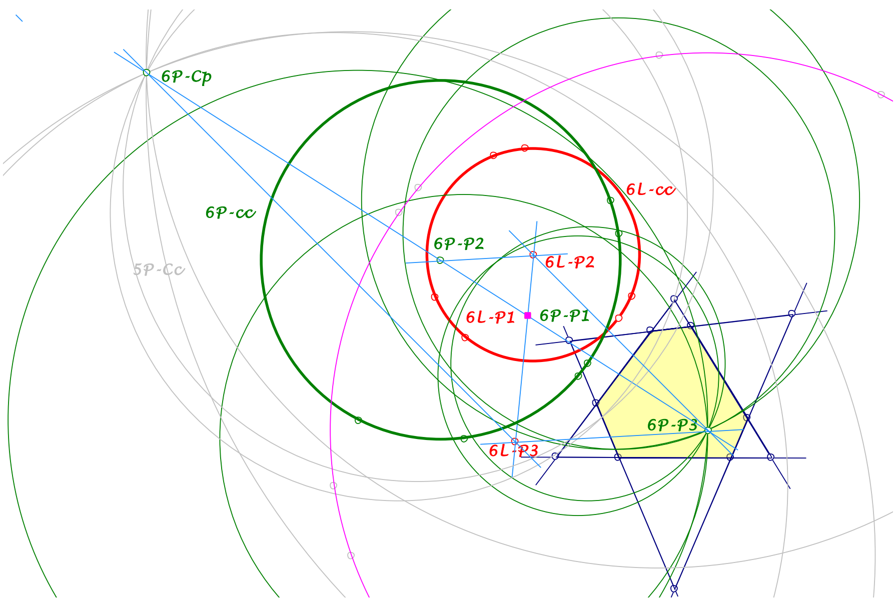
Limaçon of a 6-line

Penosculants are circles centered on a fixed circle through a fixed point, they envelope a limaçon. The center-circles *5L-cc* for 5 of 6 lines are penosculants, their envelope is the limaçon of the 6-line.

- The circle of inversion for the limaçon is centered in *6L-P1*, containing *6L-P3*.
- The vertices of the limaçon are the reflections of *6L-P3* in the intersections of *6L-P1.6L-P2* and *6L-cc*.



6P-cc, 6P-P1, 6P-P2, 6P-P3, 6P-Cp

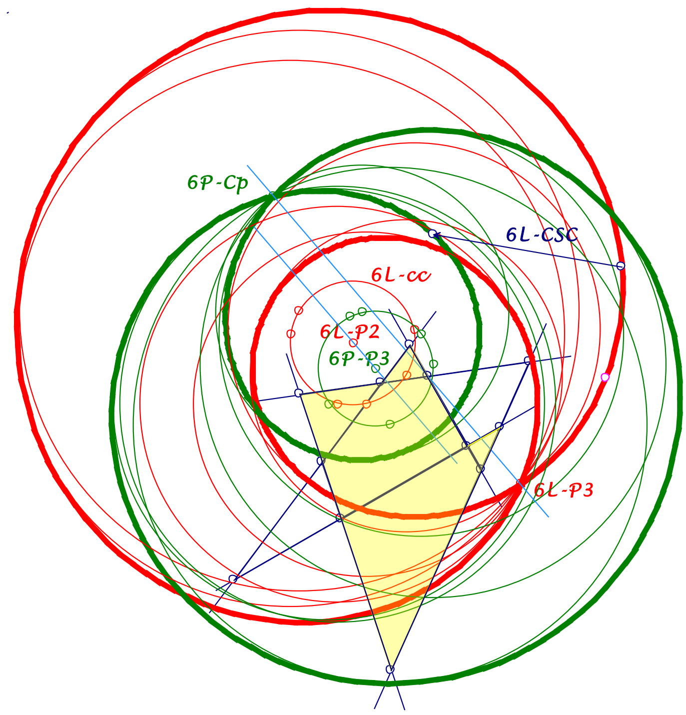


- The inverse of *6P-P3* wrt *6P-cc* is *6L-P1* = *6P-P1*.
- *6L-P2.6P-P2* is parallel *6L-P3.6P-P3*.
- *6L-P3.6P-Cp* is parallel *6L-P2.6P-P3*.
- The centers of the Clifford circles *5P-Cc* are concyclic on a circle round *6P-P3* so that *6P-P1* and *6P-Cp* are inverses.

6L-CSC-Involution

In *QFG* message *922* is described an analogon to the *CSC* transformation *QL-Tf1* for quadrilaterals. This involution can be generalized for *2n*-lines.

- The *6L-CSC*-involution has the center *6L-P1* = *6P-P1* and swaps *6L-P3* and *6P-Cp*.
- The *6L-CSC*-involution maps the *5L-cc* to the corresponding *5P-Cc*.
- The penosculant-configurations of the *5L-cc* and the *5P-Cc* for a 6-line give two limaçons, one *6L-CSC*-image of the other.



Further Involutions

The point *6L-P1* = *6P-P1* can be considered as center of different involutions. Some examples:

…partners: *6L-P2* ↔ *6L-P3*: *6L-cc* ↔ *6L-cc*,
…partners: *6P-P2* ↔ *6P-P3: 6P-cc* ↔ *6P-cc,*
... partners: *6L-P2* ↔ *6P-P3 or 6L-P3* ↔ *6P-P2:*
5L-P2 ↔ corresponding *5P-P2* (*6L-cc* ↔ *6P-cc*),
…partners: *6L-P3* ↔ *6P-P3: 6L-P2* ↔ *6P-Cp,*
5L-P2 ↔ center of the corresponding *5P-Cc.*

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