EQF-Note 2015-03-28

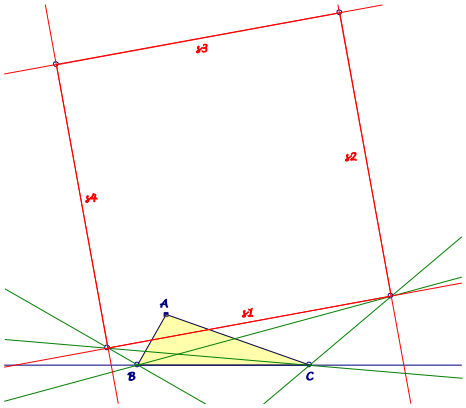
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

12 Special Morley-Axes for a Quadrilateral

Morley describes in his paper "Extensions of Clifford's Chain-Theorem" for a 4-line 64 axes. Their directions are well known, but Morley doesn't mention further properties. In his paper "64 axes of the QL" Bernard Keizer gives an interpretation and a construction of these axes (see QFG-message 1032). Here 12 special axes – out of the construction of Bernard Keizer – are considered and CABRI-tested.

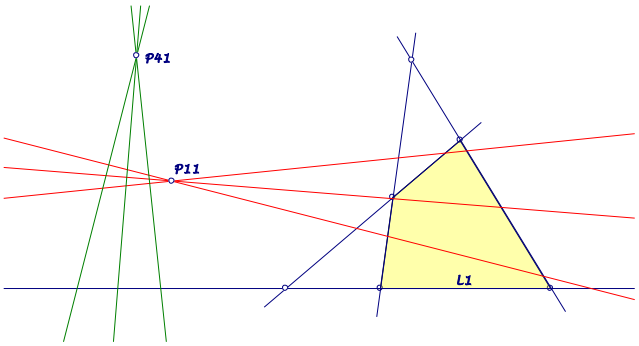
A special square for a triangle wrt a basic line

Three lines of a quadrilateral give a triangle *ABC*; let *BC* be the basic line. Morley mentioned wrt the axes, that only quadrisecting of angles is necessary. Consider the quadrisectors in *B* and *C* near to the basic line and the two intersections of an inner and an outer quadrisector. These two points are vertices of a square with circumcircle through *B* and *C*. Let s_1, s_2, s_3, s_4 be the sidelines of this square.



The 12 special Morley-axes

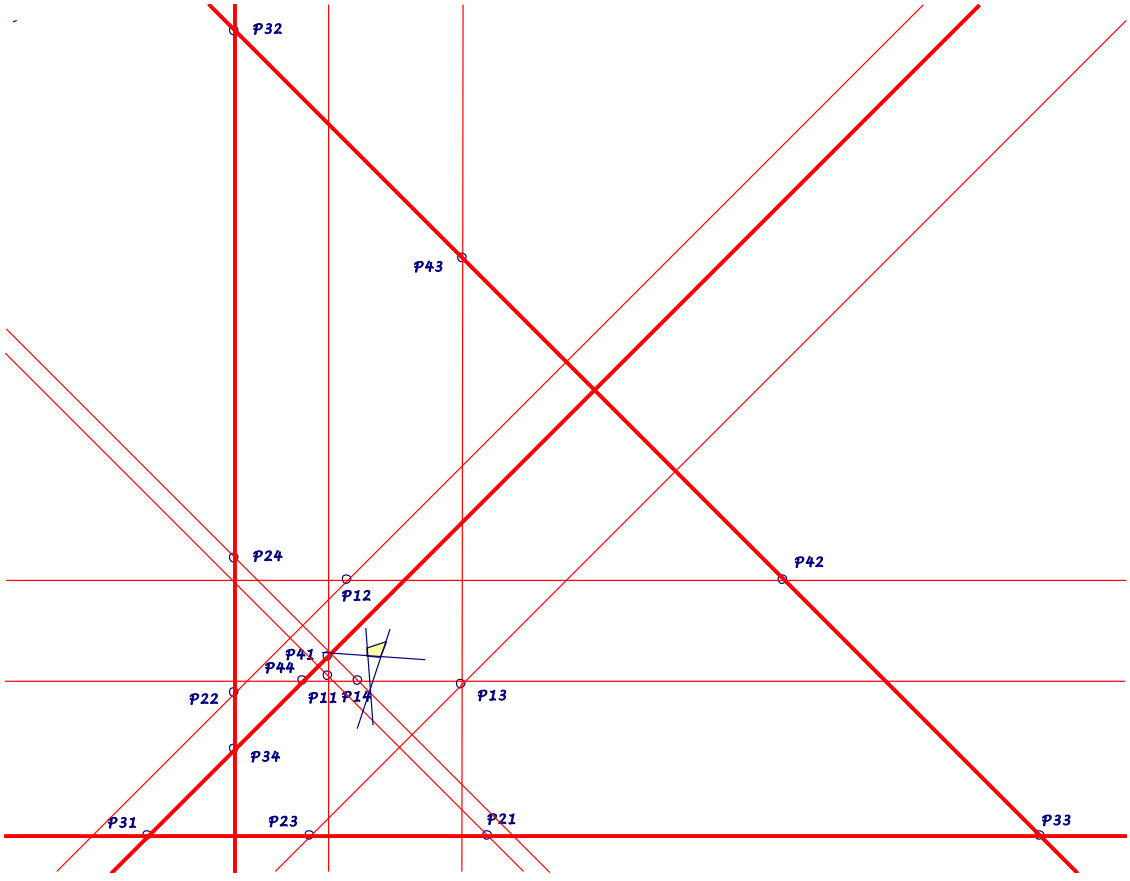
If we consider for a quadrilateral three triangles with the same basic line L_j, their lines $s_{i,j}$ have a common point $P_{i,j}$.



So we get 16 points $P_{i,j}$ and the following 12 lines (first 16, but 4 are counted twice):

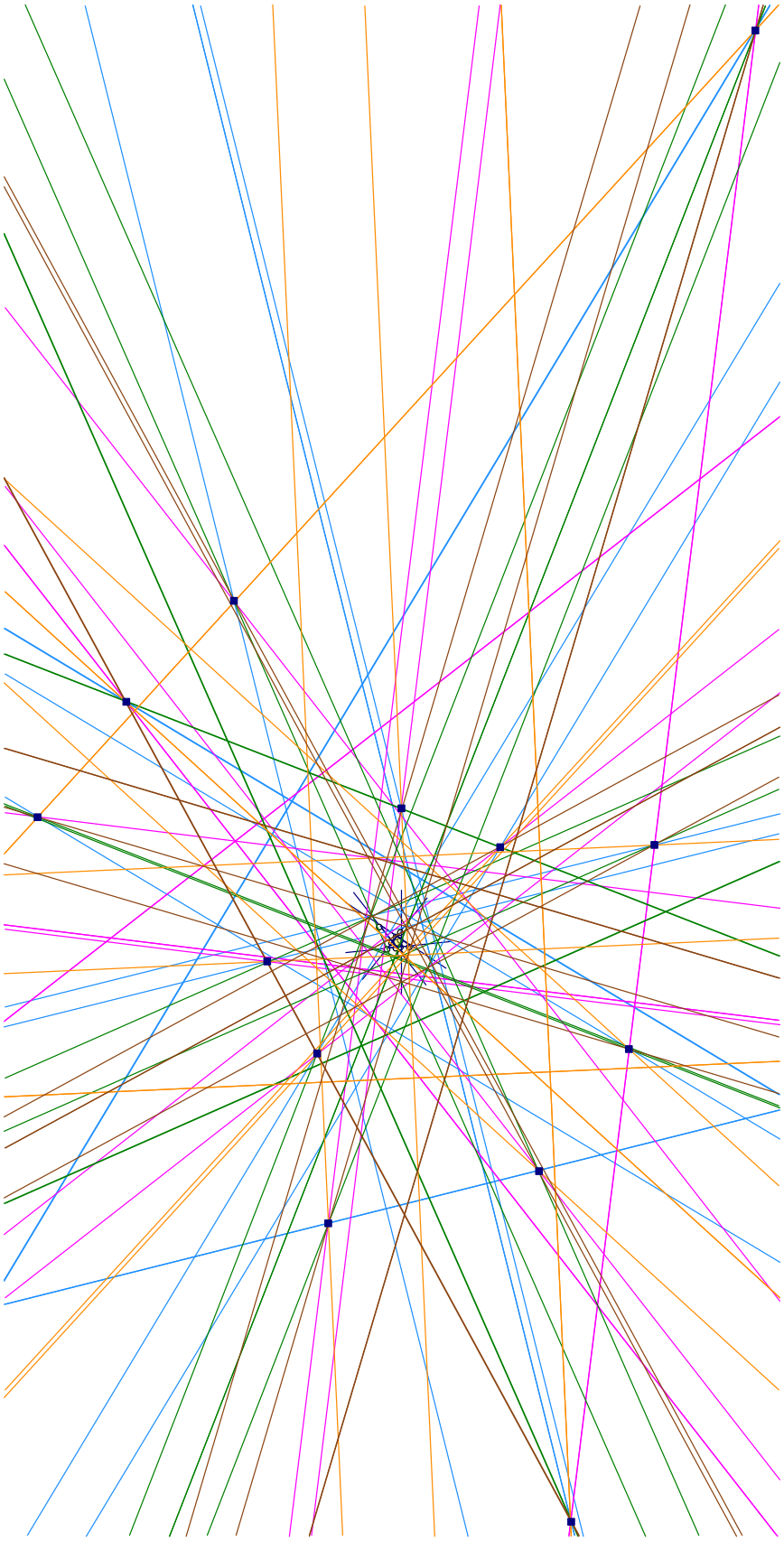
$$P_{1,j}P_{2,j},\ P_{2,j}P_{3,j},\ P_{3,j}P_{4,j},\ P_{4,j}P_{1,j} \quad \text{for } j=1,2,3,4.$$

These lines are four sets of three parallels, intersecting with angles equal modulo 45°. Further their directions satisfy Morley's condition "…clinant of an axis is a geometric mean of the clinants of the n lines. …". In accord with Bernard Keizer's construction these lines are Morley-axes of the quadrilateral.



The 4L-axes for a 5-line

For a 5-line Morley describes 4^4 "incenters" as intersections of the 5*64 axes for the five 4-lines: In our case there are 5*12 such axes for a 5-line. The corresponding constellation shows in Morley's sense five special axes with four points, where five axes intersect. These 15 intersections, where five axes intersect, will be special "incenters" of the 5-line.



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