EQF-Note 2015-04-14

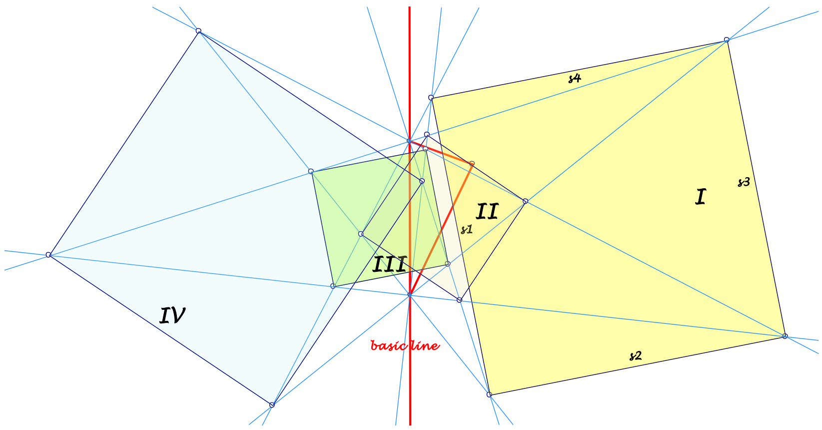
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Sets of Morley's 4L-Axes and 5L-Incenters

Morley describes in his paper "Extensions of Clifford's Chain-Theorem" for a 4-line 64 axes. Their directions are well known, but Morley doesn't mention further properties. In his paper "64 axes of the QL" Bernard Keizer gives an interpretation and a construction of these axes (see QFG-message 1032), using four squares related to quadrisectors. Each square gives a set of 4L-axes and for a 5-line a set of 5L-incenters in Morley's sense.

The squares for a triangle wrt a basic line

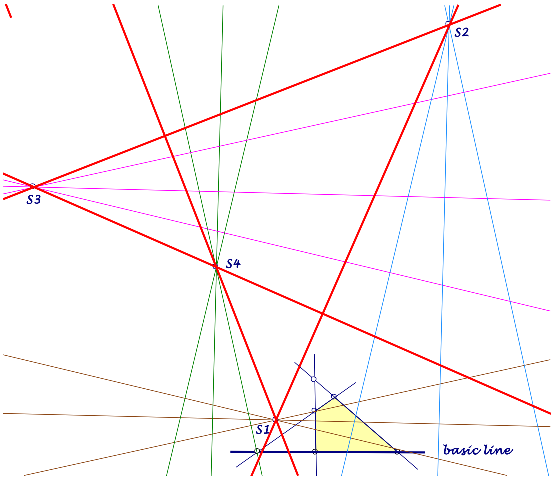
For a triangle with one side as basic line there are four squares, generated by quadrisectors and described in *QFG*-messages 1032 and 1066.



Set Ia and set Ib wrt the square I

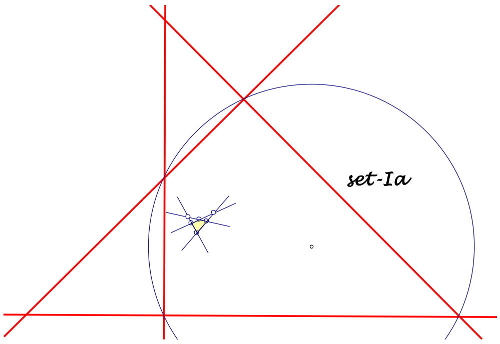
Square I shall be the first with special properties (see *QFG*-message 1054).

- If we consider for a quadrilateral three triangles with the same basic line, their lines s_i have a common point S_i.
- The lines S_1S_2, S_2S_3, S_3S_4, S_4S_1 are Morley-axes, satisfying Morley's condition "…clinant of an axis is a geometric mean of the clinants of the n lines …".



- Wrt the four basic lines of a quadrilateral, there are 12 axes (first 16, but four double counted): four sets of three parallels.

Definition: These 4 double axes give set Ia.
The 8 single axes give set Ib.



Sets wrt the other squares

Now let s_i be the side lines of another square. We consider once more three triangles of a quadrilateral with the same basic line, but their lines s_i have not a common point. Taking a sideline s_i for the three triangles, there are three intersections, giving an obtuse angled triangle. We take only the vertices X_i and Y_i with acuted angles.

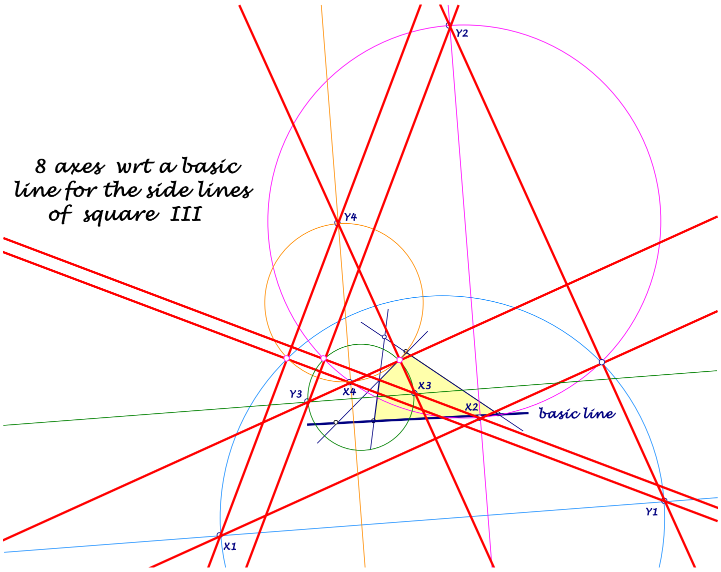
- The vertices X_i and Y_i are intersections of Morley-axes.

Let X_{i+1}, Y_{i+1} the corresponding elements wrt the next sideline s_{i+1}.

- The lines X_iY_i ($i=1,2,3,4$) give a square.

- The lines X_iX_{i+1} and Y_iY_{i+1} are orthogonal Morley-axes.

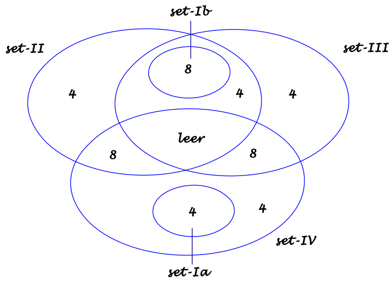
- In this way we get 8 Morley axes wrt one basic line.



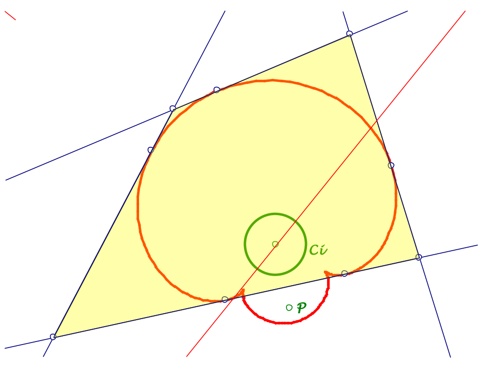
- Wrt the four basic lines of a quadrilateral there are 24 Morley axes (first 32, but 8 counted twice).

Definition:
These 24 axes for each other square give set II, set III, set IV.

- Set II, III and IV give 44 of 64 Morley-axes. Set IV contains set Ia. Set Ib is part of set II and III. The intersecting set of set II, set III and set IV is vacant.



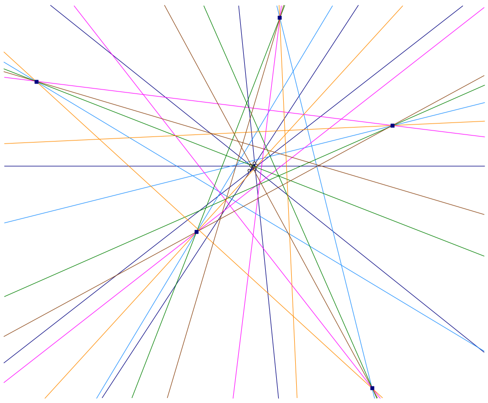
Remark: The discussed points S_i, X_i, Y_i for a quadrilateral wrt a basic line are centers of Morley's tetracardioids, tangent to the sidelines, twice for the basic line (see interpretation of Bernard Keizer in *QFG*-message 1050). A tetracardioid can be defined with a point and a circle (see *QFG*-message 831), the midpoint of the circle is the center.



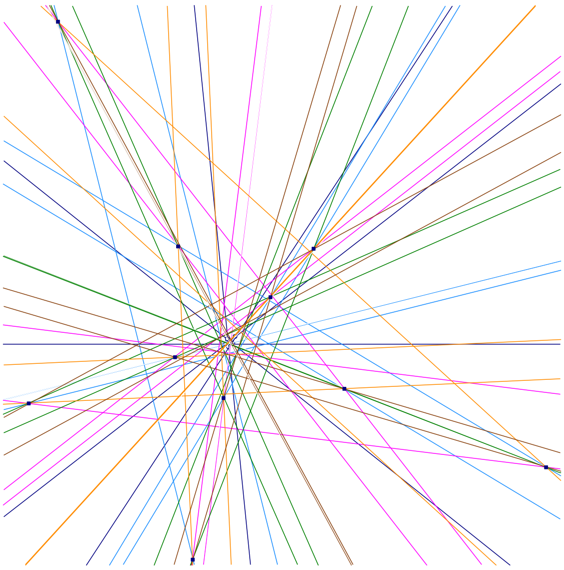
Sets of Morley-incenters for a 5-line

If we consider the axes of a quadrilateral for the five 4-lines of a 5-line, then there are 256 intersections of five, four on each axis. These are Morley-incenters of a 5-line. For the described sets of 4L-axes there are sets of 5L-incenters. Some examples:

- The set Ia (4 axes) of the five 4-lines give a constellation with five intersections of five (see figure).



- The 8 double axes of set-IV (set IV without set II and set III) for the five 4-lines give 10 intersections of five (see figure).



- Set Ia and Ib (12 axes) of the five 4-lines give 15 intersections of five (see figure in *QFG*-message 1054).

- Set IV (24 axes) of the five 4-lines give 25 intersections of five.

- Set Ia, Ib, II, III, IV (all considered 44 axes) for the five 4-lines give 92 intersections of five.

The last constellation for a 5-line contains 5*44 = 220 axes, 20 sets of 11 parallels. Every set of 11 parallels has 2 lines with 4 points, 6 lines with 2 points and 3 lines with one point, that are 23 points. 20 sets of 23 points give 92 points counted 5 times.

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