EQF-Note 2015-04-16

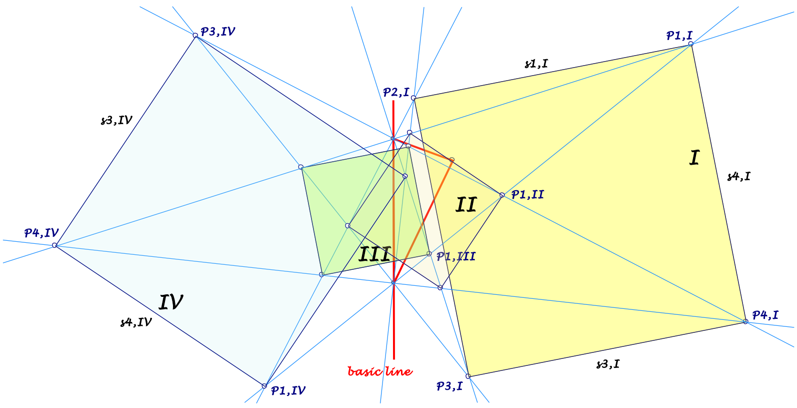
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Construction of Morley's 4L-axes

Morley describes in his paper "Extensions of Clifford's Chain-Theorem" for a 4-line 64 axes, but Morley doesn't mention a construction. In his paper "64 axes of the QL" Bernard Keizer gives an interpretation and a construction of these axes (see QFG-message 1032), using four squares related to quadrisectors. The following is only a concrete procedure for this construction.

The squares for a triangle wrt a basic line

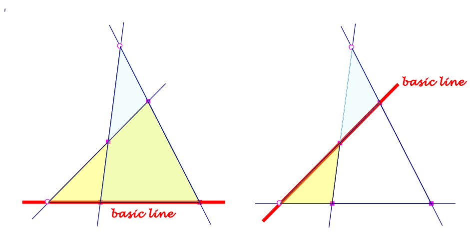
(1) For a triangle with one side as basic line there are four squares, generated by quadrisectors and described in *QFG*-messages 1032 and 1066.



(2) Chose an anticlockwise nomination for the vertices of the squares:

$P_{i,J}$ with $i = 1, 2, 3, 4$ and $J = I, II, III, IV$,

so that for example $P_{1,J}$ for $J = I, II, III, IV$ lie on the same quadrisector. Let $s_{i,J} = P_{i,J}P_{i+1,J}$ be the side lines of the squares.



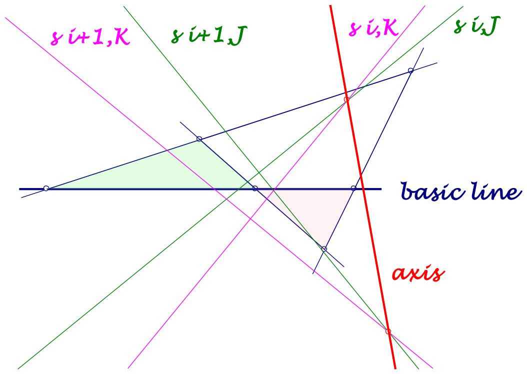
(3) Consider for a quadrilateral a basic line. There are three triangles with this basic line, but we take only two: If

the three triangles lie on one side of the basic line, we use the two, which intersect in the convex quadrigon of the quadrilateral. If the three triangles lie on different sides of the basic line, we use the two, which have no intersection with the convex quadrigon of the quadrilateral.

(4) Take for a basic line of a quadrilateral the two triangles in the sense of (3). Draw for one triangle $s_{i,J}$ and $s_{i+1,J}$, for the other $s_{i,K}$ and $s_{i+1,K}$, then:

$s_{i,J} \cap s_{i,K} \cdot s_{i+1,J} \cap s_{i+1,K}$ **is a Morley-axis**

$i \in \{1, 2, 3, 4\}$, J and $K \in \{I, II, III, IV\}$.



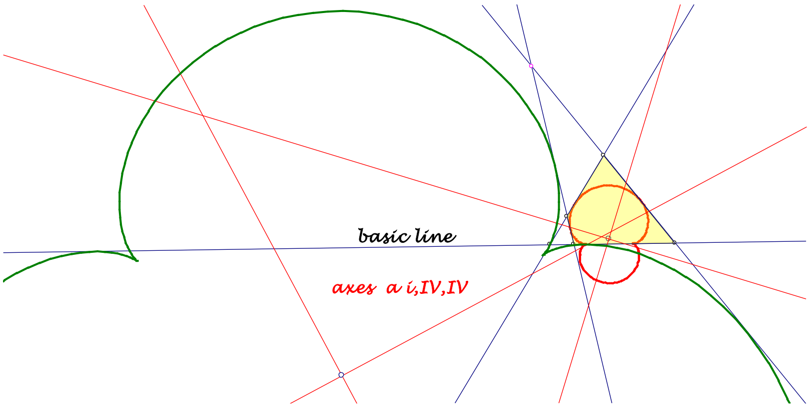
Remarks:

- Let $\boldsymbol{a}_{i,J,K}$ be the so constructed axis wrt a basic line, then the four possibilities for i, J, K show, that there are 4^3 = 64 Morley-axes for a quadrilateral.

- For $J = K$ wrt a basic line there are four axes, that gives wrt four basic lines 16 axes, but four are counted twice. For $J = K = I$ these 12 axes are described in *QFG*-message 1054, further in *QFG*-message 1067 as set Ia and set Ib. For $J = K = II, III, IV$ these axes give set II, III, IV in *QFG*-message 1067.

- The points $s_{i,J} \cap s_{i,J}$ are intersections of axes, which will be the centers for Morley's tetracardioids, tangent to the sidelines of the quadrilateral, twice for the basic line.

- Final figure as Cabri observation to prove: The four axes $\boldsymbol{a}_{i,IV,IV}$ wrt a basic line are two pairs of orthogonal axes. The intersection of each pair is a center of a tetracardioid, tangent to the four sides and twice to the basic line in symmetry.



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