EQF-Note 2015-04-22

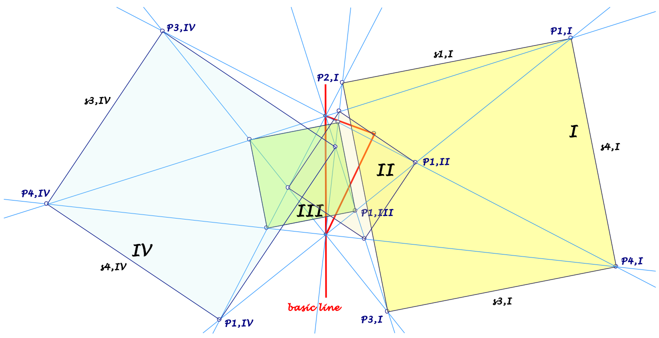
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Orthogonal Morley-4L-axes

Morley describes in his paper "Extensions of Clifford's Chain-Theorem" for a 4-line 64 axes, but Morley doesn't mention a construction. In his paper "64 axes of the QL" Bernard Keizer gives an interpretation and a construction of these axes (see QFG-message 1032), using four squares related to quadrisectors and 96 points wrt a basic line. Here the constellation of 32 of these points is researched, where Morley-axes intersect orthogonal.

The squares for a triangle wrt a basic line

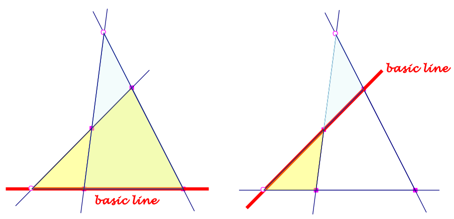
(1) For a triangle with one side as basic line there are four squares, generated by quadrisectors and described in *QFG*-messages 1032 and 1066.



(2) Chose an anticlockwise nomination for the vertices of the squares:

$P_{i,J}$ with $i = 1, 2, 3, 4$ and $J = I, II, III, IV$,

so that for example $P_{1,J}$ for $J = I, II, III, IV$ lie on the same quadrisector. Let $s_{i,J} = P_{i,J}P_{i+1,J}$ be the side lines of the squares.



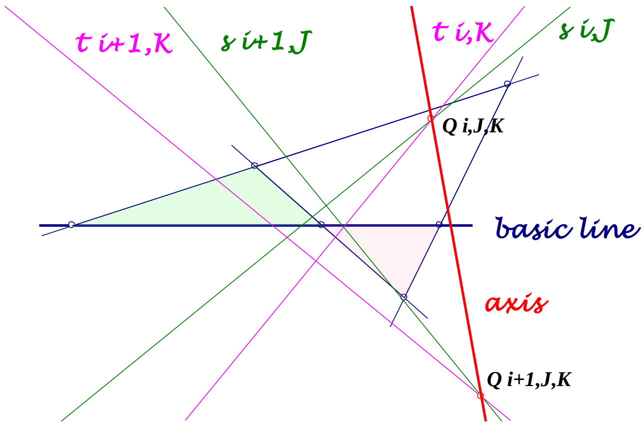
(3) Consider for a quadrilateral a basic line. There are three triangles with this basic line, but we take only two: If

the three triangles lie on one side of the basic line, we use the two, which intersect in the convex quadrigon of the quadrilateral. If the three triangles lie on different sides of the basic line, we use the two, which have no intersection with the convex quadrigon of the quadrilateral.

(4) Take for a basic line of a quadrilateral the two triangles in the sense of (3), distinguished by $\boldsymbol{s}$ and $\boldsymbol{t}$. Draw for one triangle $\boldsymbol{s}_{i,J}$ and $\boldsymbol{s}_{i+1,J}$, for the other $\boldsymbol{t}_{i,K}$ and $\boldsymbol{t}_{i+1,K}$ and consider the intersections $\boldsymbol{s}_{i,J} \cap \boldsymbol{t}_{i,K} = \boldsymbol{Q}_{i,J,K}$ and $\boldsymbol{s}_{i+1,J} \cap \boldsymbol{t}_{i+1,K} = \boldsymbol{Q}_{i+1,J,K}$.

$\boldsymbol{Q}_{i,J,K}\,\boldsymbol{Q}_{i+1,J,K}$ is a Morley-axis

$i \in \{1, 2, 3, 4\}$, J and $K \in \{I, II, III, IV\}$.



- The four possibilities for i, J, K show, that there are 4^3 = 64 Morley-axes wrt a basic line for a quadrilateral. The four basic lines give the same Morley-axes.

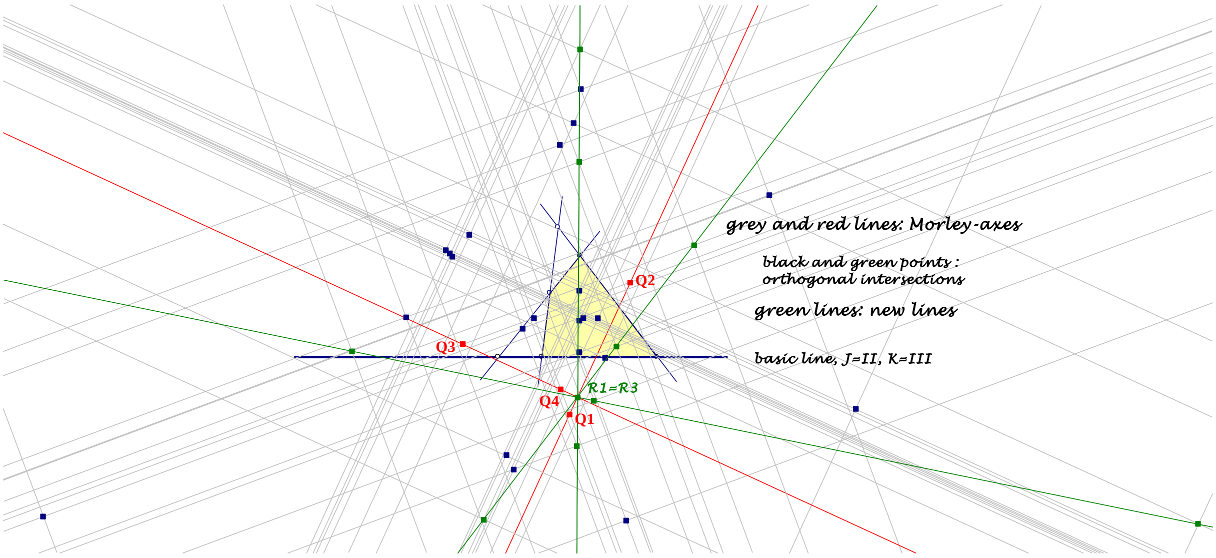
- The 64 points $\boldsymbol{Q}_{i,J,K}$ are intersections of the Morley-axes $\boldsymbol{Q}_{i-1,J,K}\,\boldsymbol{Q}_{i,J,K}$ and $\boldsymbol{Q}_{i,J,K}\,\boldsymbol{Q}_{i+1,J,K}$.

- The Morley-axes $\boldsymbol{Q}_{i,J,K}\,\boldsymbol{Q}_{i+1,J,K}$ and $\boldsymbol{Q}_{i+2,J,K}\,\boldsymbol{Q}_{i+3,J,K}$ are orthogonal.

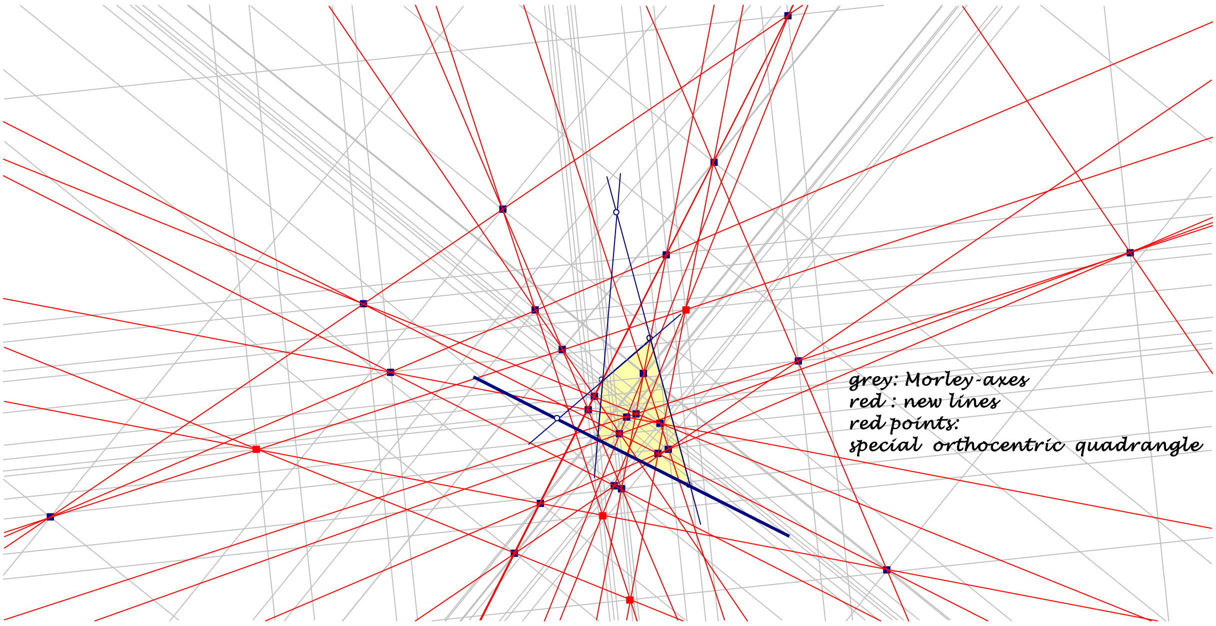
- There are 32 intersections $\boldsymbol{R}_{i,J,K} = \boldsymbol{R}_{i+2,J,K} = \boldsymbol{Q}_{i,J,K}\,\boldsymbol{Q}_{i+1,J,K} \cap \boldsymbol{Q}_{i+2,J,K}\,\boldsymbol{Q}_{i+3,J,K}$ of such orthogonal Morley-axes.

- If we consider also the diagonals $\boldsymbol{s}_{\alpha,J} = \boldsymbol{P}_{1,J}\boldsymbol{P}_{3,J}$ and $\boldsymbol{s}_{\beta,J} = \boldsymbol{P}_{2,J}\boldsymbol{P}_{4,J}$ of the squares, we can get the points $\boldsymbol{R}_{i,J,K}$ as $\boldsymbol{s}_{\alpha,J} \cap \boldsymbol{t}_{\alpha,K}$ for $i = 1or3$ and as $\boldsymbol{s}_{\beta,J} \cap \boldsymbol{t}_{\beta,K}$ for $i = 2or4$.

Among Bernhard Keizer's 96 points of a 4-line (wrt a basic line) these 32 intersections $\boldsymbol{R}_{i,J,K}$ give a very special constellation:



... Sets of 4 of these points are collinear (green).
... The corresponding lines intersect by 3 in one of these points.
... These are 24 new lines: 12 pairs of parallels, each pair with a perpendicular pair.
… The constellation contains in different ways orthocentric quadrangles, their lines bear 16 of the 32 points.
... For the 4 points on a bearer line the angles between the other two lines are equal.



Bernhard Keizer's 96 points of a 4-line are the centers of Morley's tetracardioids, tangent to the four lines and twice to the basic line.

The 32 special orthogonal intersections $R_{i,J,K}$ of Morley-axes are the centers of tetracardioids tangent to the four lines and twice in symmetry to the basic line.

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