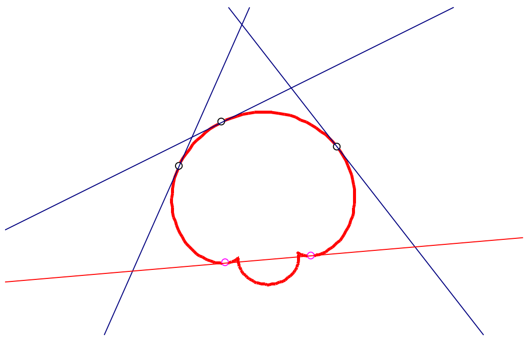
EQF-Note 2015-04-28

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

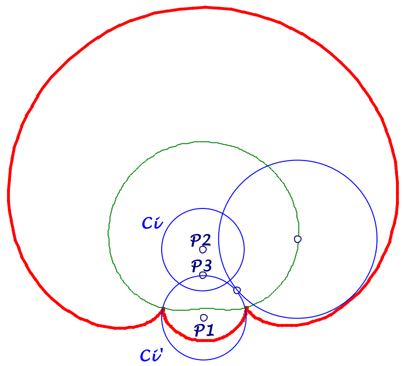
Double Tangent Tetracardioïds

Morley mentioned in his paper "Extensions of Clifford's Chain-Theorem" for a 4-line 96 tetracardioïds tangent to the four lines and to one line twice. Centers of these tetracardioïds are special intersections of Morley's 64 axes for a 4-line. Here a set of 32 tetracardioïds is researched, which contact a line twice in symmetry.



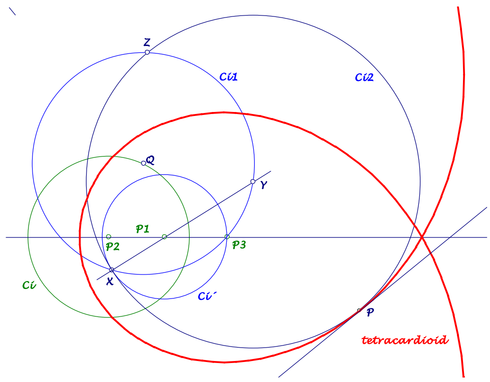
Tetracardioïds

A limaçon is the envelope of circles centered on a circle and through a fixed point. A tetracardioïd can be described as envelope of circles centered on a limaçon and tangent to a circle. Let P_1 be a point, Ci a circle with center P_2 and P_3 the inverse of P_1 wrt Ci. Consider the limaçon wrt Ci and P_3 and a circle Ci' round P_1 through P_3. Circles centered on the limaçon and tangent to Ci' envelope a tetracardioïd. If P_1 lies inside Ci, let Ci' be inside the contacting circles, if P_1 lies outside Ci, let Ci' be outside the contacting circles.

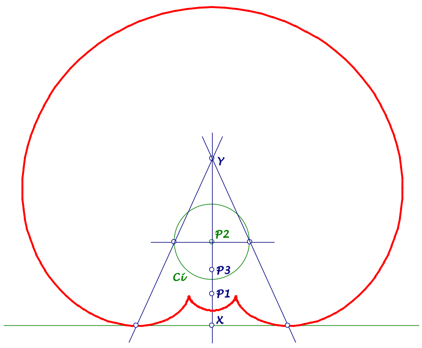


In this way a tetracardioïd is defined by a point P_1 and a circle Ci. P_2 is the center of the tetracardioïd. If P_1 lies inside the circle, the tetracardioïd is limaçon-similar, if P_1 lies outside the circle the tetracardioïd has two cusps. A construction can be done as follows:

... Let Q be a variable point on the circle Ci,
... let Ci_1 be a circle round Q through $P3$,
... let X be the second intersection of Ci_1 and the circle $Ci´$,
... let Y be the second intersection of XP_1 and Ci_1,
... let Ci_2 be a circle round Y through X,
... let Z be the second intersection of Ci_1 and Ci_2,
... then Z reflected in Y is a point P of the tetracardioïd
…with the same tangent to Ci_2.



Tetracardioïd with a given double tangent



We consider only double tangents in symmetry. If the double tangent is given, every circle Ci with center P_2 gives a corresponding tetracardioïd, constructed as follows:

… Let X be the pedal point of P_2 wrt the double tangent,
… let P_3 divide P_2X in the ratio 1:2,
… let P_1 be the inverse of P_3 wrt Ci,
… further construction see above.

The contact points can be constructed as follows:

… Let Y be the reflection of X in P_2,
… let S_1 and S_2 be the intersections of a perpendicular line in P_2 wrt XY,
… then YS_1 and YS_2 intersect the double tangent in the contact points.

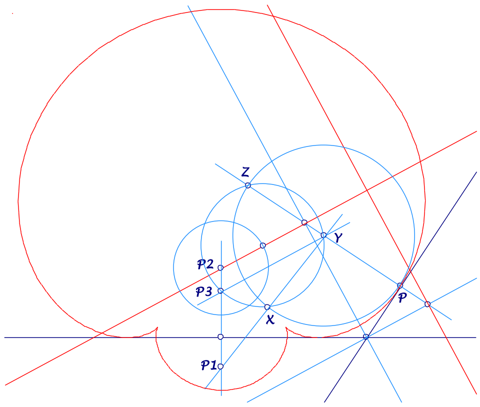
Tetracardioïds with a given double tangent and a 2nd tangent

Let the double tangent be given and a 2nd tangent with contact point *P*. The centers of corresponding tetracardioïds lie on two lines:

… Let *u* and *u´* be the angle bisectors of the given tangents,

… let *v* be a perpendicular in *P* wrt the 2nd tangent,

… then the perpendiculars wrt the angle bisectors in the intersections with *v* are the loci for centers of tetracardioïds contacting the 2nd tangent in *P*.



If we chose a center P_2 on these lines, the tetracardioïd can be constructed in a way back:

… Let P_3 divide the distance between P_2 and its pedal point on the double tangent with ratio 1:2,

… a parallel to the locus line of the centers through P_3 intersects the perpendicular in *P* in the point *Y*,

…let Ci_2 be a circle round *Y* through *P* and *Z* the reflection of *P* in *Y*,

… let Ci_1 be the circumcircle of *Y*, *Z*, P_3 with second intersection *X* with Ci_2,

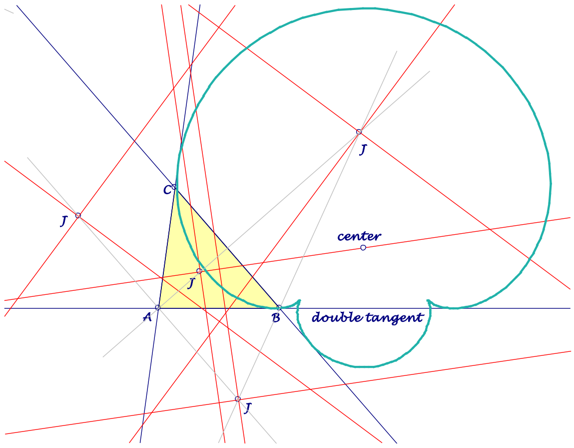
… the line *XY* intersects the perpendicular through P_2 wrt the double tangent in P_1,

... then the circle *Ci* for the tetracardioid is a circle round P_2 so that P_1 and P_3 lie inverse.

Tetracardioïds with a given double tangent and two other tangents

This constellation gives a triangle *ABC*: Let *AB* be the double tangent and *AC*, *BC* two tangents. The described properties and constructions give the following CABRI observation:

- **Let *J* be an in- or excenter of *ABC*, then the angle bisectors of <*AJB* are the loci for centers of tetracardioïds tangent to *AC*, *BC* and double tangent to *AB* in symmetry.**

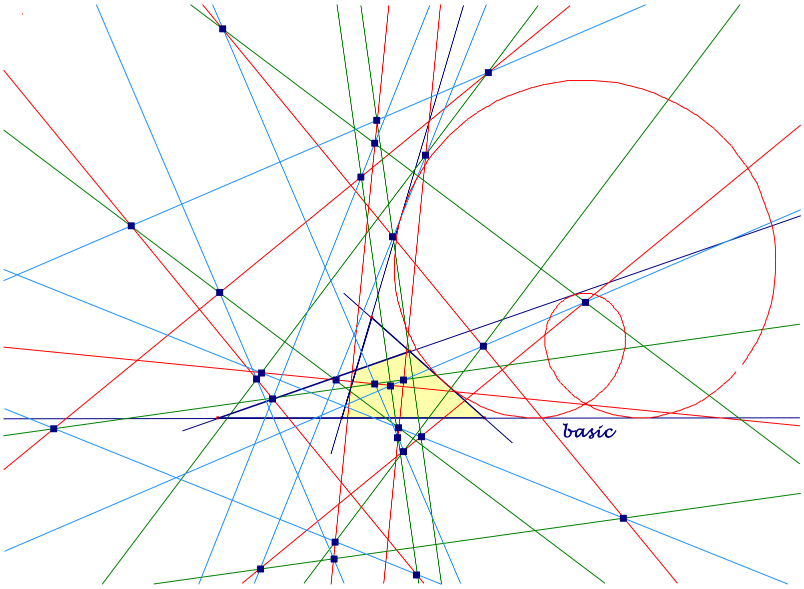


There are 8 center lines: 4 pairs of parallels or 4 pairs of orthogonal lines, intersecting in the in- and excenters.

Tetracardioïds with a given double tangent and three other tangents

Background now is Morley's constellation for 96 inscribed tetracardioïds of a 4-line, double tangent to one line. But here are only considered the double contacts in symmetry. If one line of the 4-line is declared as double tangent or basic line, there are three triangles with this side line.

- **The centers of inscribed tetracardioïds for a 4-line, double tangent to a basic line in symmetry, are the 32 triple intersections of the 8 center lines for the 3 triangles at the basic line.**



On each center line of the three triangles are 4 centers of tetracardioïds. This constellation is already described in *EQF*-message 1083 with the result:

- **The 32 centers of inscribed tetracardioids of a 4-line, double tangent to a basic line in symmetry, are the intersections of orthogonal Morley-axes among Bernard Keizer's 96 points of a 4-line wrt a basic line.**

Wrt to 4 possible basic lines for a 4-line there are 96 centers of inscribed tetracardioïds, tangent twice to one side in symmetry.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de