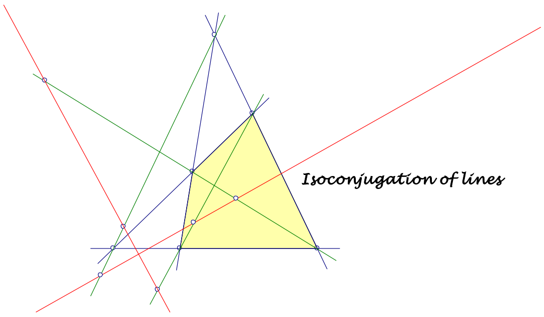
EQF-Note 2015-06-20

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Tf2 and QL-Cu1

The isoconjugation for lines QL-Tf2 has rarely feedback in EQF, but there are interesting properties up to cubics as QL-Cu1. Here is a short summary of my paper [1], which gives barycentric calculations for the following results.



The Involutary Conjugate *QA-Tf2* is an isoconjugation for points wrt the *QA*-Diagonal Triangle *QA-Tr1*. The *QL*-Line Isoconjugate *QL-Tf2* is its analogon for lines wrt the *QL*-Diagonal triangle *QL-Tr1*. The *QL-Tf2*-image of a line is the locus of its poles wrt the inscribed conics of the quadrilateral.

But here we start with a simple construction for *QL-Tf2*, based on a theorem of Chasles [2]:

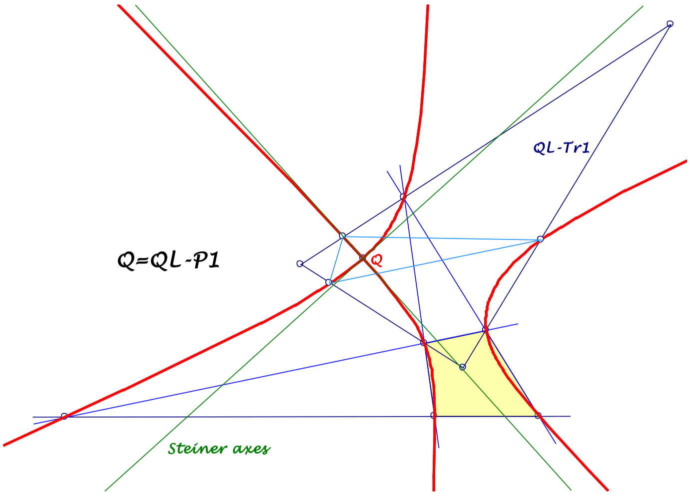
- **Let a line intersect the diagonals of a quadrilateral, then the 4th harmonic points on the diagonals are collinear.**

This line transformation *QL-Tf2* has the lines of the quadrilateral as fixed lines. The *QL-Tf2*-image of the Newton line is the line at infinity. The *QL-Tf2*-image of *QL-L2* is a parallel to *QL-P3.P4.P5.P6* through *QL-P1*. The Steiner axes are *QL-Tf2*-partners.

- **The *QL-Tf2*-images of lines through a fixed point envelope an inscribed conic of the *QL*-Diagonal Triangle *QL-Tr1*.**

For *QL-P13* we get the inscribed Steiner ellipse of *QL-Tr1*. For points on the Newton line these conics are inscribed parabolas of *QL-Tr1*. For the Miquel point *QL-P1* we get a special inscribed conic of *QL-Tr1*, which contacts the Steiner axes and the line *QL-L2* (see *QFG*-message 481).

- **The intersections of lines through a fixed point *Q* and their *QL-Tf2*-image give a cubic through …
  … the six vertices of the quadrilateral,
  … the vertices of the Ceva triangle of *Q* wrt *QL-Tr1*.**



Example: Let *Q* be the Miquel point *QL-P1*, which will be a knot of the cubic with the Steiner axes as tangents. This cubic is invariant wrt an isoconjugation with fixed point *QL-P1* and its Ceva triangle wrt *QL-Tr1* as reference triangle.

If *Q* is a *CSC*-fixed point *QL-2P3*, the cubic contains *QL-P1* and is *CSC*-invariant.

Of special interest are points, which have in their line pencil an orthogonal pair of *QL-Tf2*-partners. The locus for these points will be *QL-Cu1*.

- ***QL-Cu1* is the locus for points, whose angle bisectors wrt two opposite vertices are *QL-Tf2*-partners.**

Finally:

- **The intersections for *QL-Tf2*-images of perpendicular lines of a pencil are collinear.**

Example: For the line pencil of *QL-P1* we get a parallel to *QL-P3.P4.P5.P6* through *QL-P1*.

References:

[1] http://eckartschmidt.de/Isofl.pdf
[2] M. Chasles, Ann. de math. 18 (1827-1828), p. 297.

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