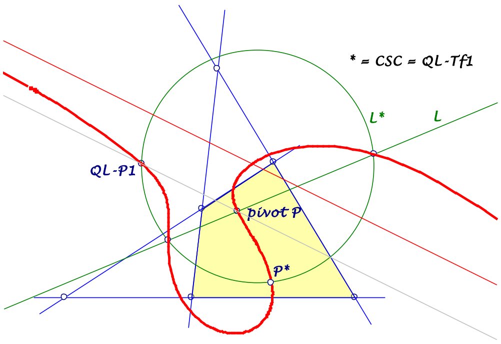
EQF-Note 2015-07-22

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Pivotal *CSC*-Cubics for a Quadrilateral

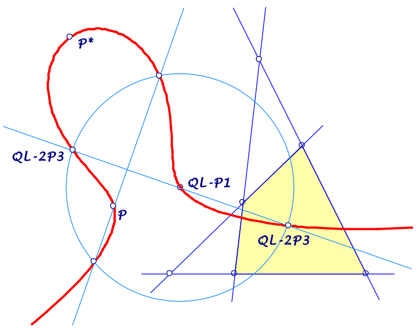
Well-known are "pivotal isocubics", defined wrt a triangle, an isoconjugation and a pivot [1]. Here an analogon is considered wrt a quadrilateral, the CSC-transformation (QL-Tf1) and a pivot. General properties are presented, but only CABRI-controlled.



A pivotal *CSC*-cubic of a quadrilateral shall be the locus for intersections of lines *L* through a pivot point *P* and their *CSC*-circles *L.**

- **A pivotal *CSC*-cubic degenerates for a pivot *P* on the Steiner Axes.**

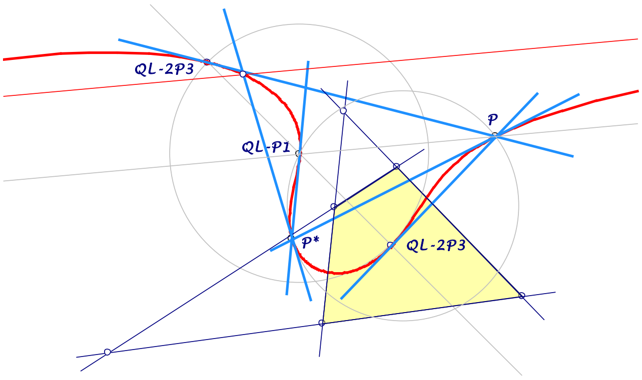
- **A pivotal *CSC*-cubic with pivot *P* contains the following points:**
  … the Miquel Point *QL-P1*,
  … the *CSC*-fixed points *QL-2P3*,
  … the pivot *P* and its *CSC*-partner *P,**
  … the intersections of the Schmidt Circle and a perpendicular line wrt the 1st Steiner Axis through *P*.



- **A pivotal *CSC*-cubic with pivot *P* is the locus for the intersections of circles through *QL-P1*, centered on the bisector of *P*.QL-P1*, and their *CSC*-lines.**

- **A pivotal *CSC*-cubic is *CSC*-invariant. *CSC*-partners are collinear with the pivot.**

- **The asymptote is a parallel to *P.QL-P1* through the reflection of *P** in *P.QL-P1*.**



- **Tangents at the cubic in the *CSC*-fixed points *QL-2P3* intersect in the pivot point *P*.**

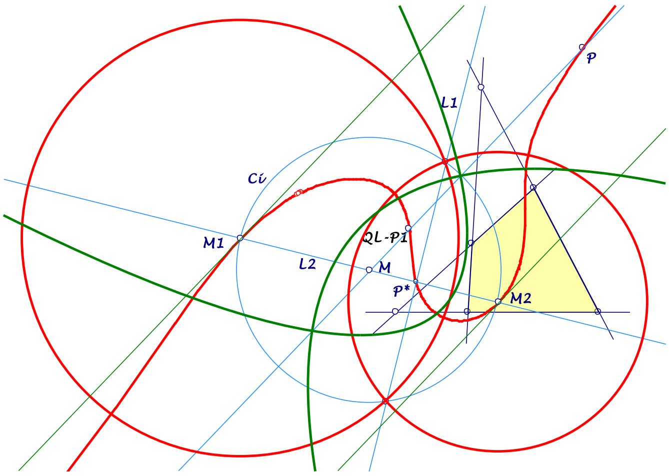
- **The tangent in *P* at the cubic is *PP**.**

- **The tangent in *P** at the cubic is the tangent in *P** at the circumcircle of *P*, *P** and *QL-P1*.**

- **The intersection of the cubic and its asymptote is the intersection with the tangent at *P**.**

- **A pivotal *CSC*-cubic is an anallagmatic curve.**

The cubic is invariant wrt two inversions. The inversion circles can be constructed as follows:

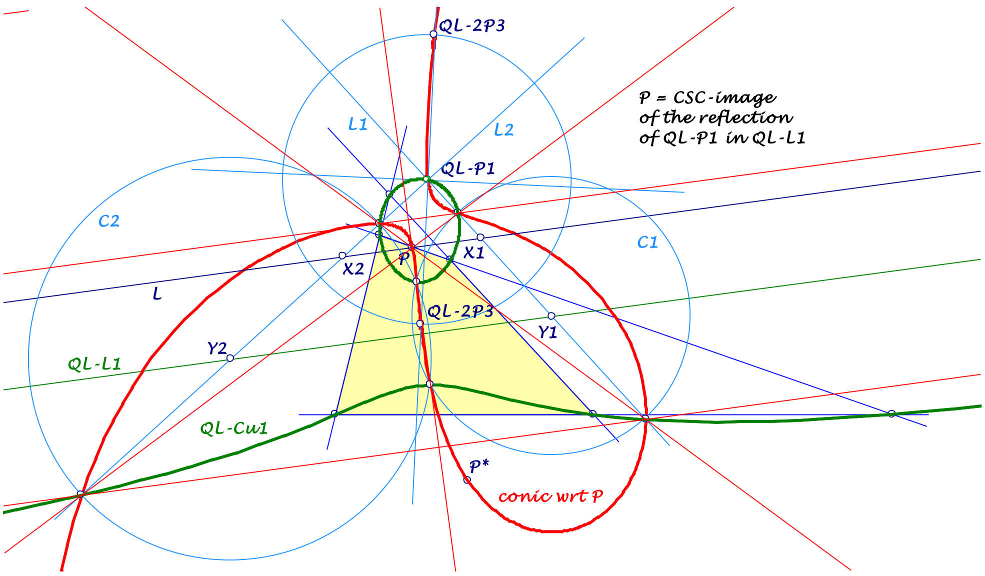


… L_1 : angle bisector at P^* wrt P, *QL-P1*,
… L_2 : perpendicular line wrt L_1 through P^*,
… M : intersection of L_2 and *P.QL-P1*,
… Ci : inversion circle round M wrt P and *QL-P1*,
… M_i : intersections of Ci and L_2.
Circles round M_i through the intersections of L_1 and Ci are inversion circles for the cubic. The tangents in M_i are parallel to the asymptote, that means parallel to *P.QL-P1*.

As anallagmatic curve the cubic is twice the envelope of circles, centered on a parabola and orthogonal to one inversion circle. The parabolas have the same focus in P^* and as directrices the tangents in M_1 and M_2.

Final example:

Taking for pivot the *CSC*-image of the reflection of *QL-P1* in *QL-L1*, the cubic is the locus for intersections of circles through *QL-P1* and centered on *QL-L1* with their *CSC*-lines. This cubic has intersections with the *QL*-Quasi Isogonal Cubic *QL-Cu1* on the angle bisectors of the Steiner Axes.
QL-P1 is one intersection of the here discussed cubic and *QL-Cu1*, but there are further three pairs of intersections, not always real. They can be constructed as follows:



… P : *CSC*-image of the reflection of *QL-P1* in *QL-L1*,
… L_1, L_2 : angle bisectors of the Steiner Axes,
… L : parallel to *QL-L1* through P,
… X_i : intersections $L \cap L_i$,
… Y_i : intersections $L_i \cap$ *QL-L1*,
… C_i : inversion circles round Y_i wrt X_i and *QL-P1* (*CSC*-partners),
… $C_i \cap L_i$: intersections of the cubics (not always real),
… $C_1 \cap C_2$: intersections of the cubics, if *QL-Cu1* is bipartite.

The first four intersections ($C_i \cap L_i$) lie in pairs on äquidistant parallels wrt *QL-L1*. The six intersections of the cubics lie by four on the circles C_i and in pairs of *CSC*-partners on lines through *P*.

References:

[1] Jean-Pierre Ehrmann and Bernard Gibert: Special Isocubics in the Triangle Plane.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de