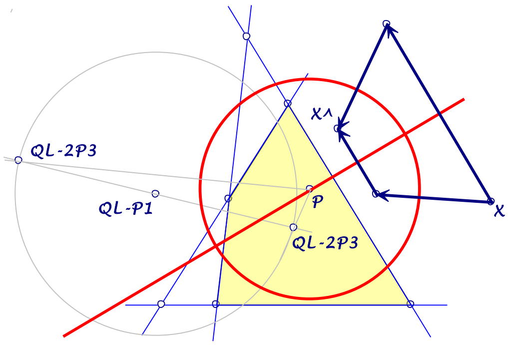
EQF-Note 2015-07-24

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Generalization of *CSC (QL-Tf1)*

*The CSC-transformation is determined by a **P**oint (Miquel Point QL-P1), a **L**ine (1st Steiner Axis) and a **C**ircle (Schmidt Circle). A generalization gives a **PLC**-transformation, which leads to pivotal QL-cubics analog to "pivotal isocubics" for a triangle, replacing the isoconjugation by PLC-transformation for a quadrilateral. General properties are presented, but only CABRI-controlled.*



PLC-Transformations

A *PLC*-transformation for a quadrilateral is defined wrt a point *P* as center, a line through *P* as axis and a circle round *P* as inversion circle. The construction is orientated at the *CSC*-transformation * and its fixed points *QL-2P3*.

A *PLC*-transformation wrt a center *P* is the product of … **… a reflection in the angle bisector at *P* wrt *QL-2P3a,b*** **… and an inversion wrt a circle round *P* with radius** $\sqrt{P.QL-2P3a}\sqrt{P.QL-2P3b}$ **.**

The definition fails for centers *P* = *QL-2P3*. Properties of *PLC* are similar to those of *CSC*: The center *P* has no *PLC*-image. *PLC*-image of a line *L* is a circle through *P*, degenerating in a line, if *P* on *L*. *PLC*-image of a circle *Ci* is a circle, degenerating in a line, if *P* on *Ci*.

- **The *PLC*-transformation swaps *QL-2P3a,b* and *QL-P1* with *P**.**

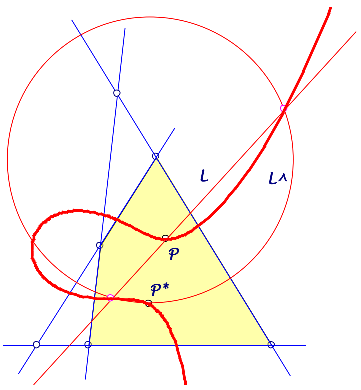
- **For center *P* = *QL-P17* holds *PLC(QL-L1)* = *QL-Ci1*.**

- **The *QL*-Quasi Isogonal Cubic *QL-Cu1* is invariant under special *PLC*-transformations …**
  … for centers in *QL-2P2* (intersections of *QL-L1* and its *CSC*-circle), if *QL-Cu1* is unipartite,
  … for centers in intersections of a perpendicular line to *QL-L1* at the intersection with *QL-L6* and its *CSC*-circle, if *QL-Cu1* is bipartite.

Pivotal *PLC*-Cubics

For pivotal *CSC*-cubics see *QFG*-message 1237. Here is an analog definition for pivotal *PLC*-cubics, using…

… **a *PLC*-transformation ^ with center *P** for the pivot *P*:**

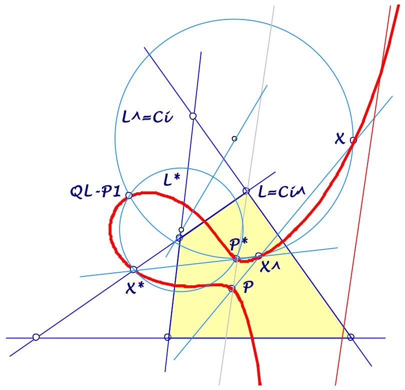


A pivotal *PLC*-cubic of a quadrilateral shall be the locus for intersections of lines *L* through a pivot point *P* and their *PLC*-circles *L*^.

The definition fails for pivots *P* on the Steiner Axes.

- **A pivotal *PLC*-cubic with pivot *P* and lines *L* through *P** is the locus for intersections of the circles *L** and the lines *L*^.**

- **Pivotal *PLC*-cubics with pivots *P* and *P** are the same.**

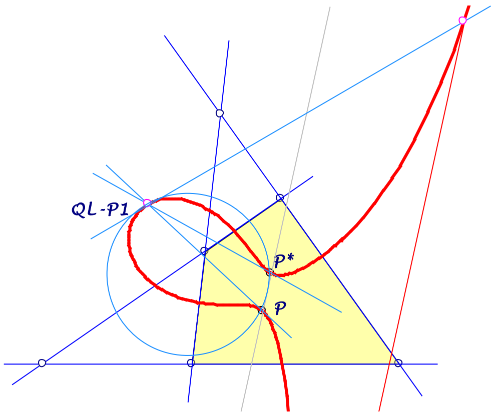


- **A pivotal *PLC*-cubic with pivot *P* is the locus for intersections of circles *Ci* through *QL-P1*, centered on the bisector of *P*.QL-P1*, and lines *Ci*^.**

- **A pivotal *PLC*-cubic with pivot *P* contains the points *P, P** and *QL-P1* with *P*^ = *QL-P1*.**

- **For points *X* on a pivotal *PLC*-cubic with pivot *P*…**
  … *X, X*^ and *P* are collinear,
  … *X*^, *X and *P** are collinear.**

- **A pivotal *PLC*-cubic with pivot *P* is invariant wrt the *CSC*-transformation and the *PLC*-transformations with center *P* and *P**.**



- **The asymptote is a parallel to *PP** through the reflection of *QL-P1* in *P* (or *P**).**

- **The tangents in *P* and *P** at the cubic intersect in *QL-P1*.**

- **The tangent in *QL-P1* at the cubic is the tangent in *QL-P1* at the circumcircle of *P, P** and *QL-P1*.**

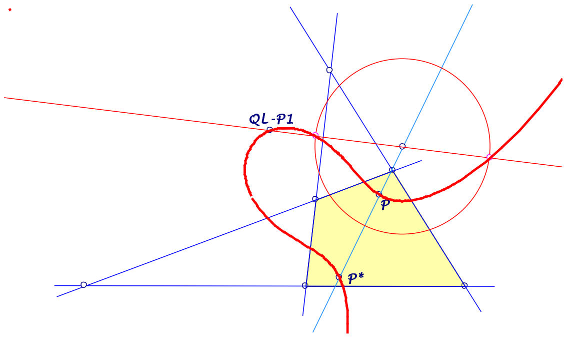
- **The intersection of the cubic and its asymptote is the intersection with the tangent at *QL-P1* (see above).**

- **The fixed points of ^ lie on the cubic as *CSC*-partners.**

- **An unipartite *QL*-Quasi Isogonal Cubic *QL-Cu1* is a pivotal *PLC*-cubic for pivots in *QL-2P2*.**

The fact, that *P* and *P** as pivot give the same cubic leads to a simple construction without *PLC*-transformation:

- **Inversion circles for the pivots *P* and *P** cut the lines from their center to *QL-P1* on the cubic.**

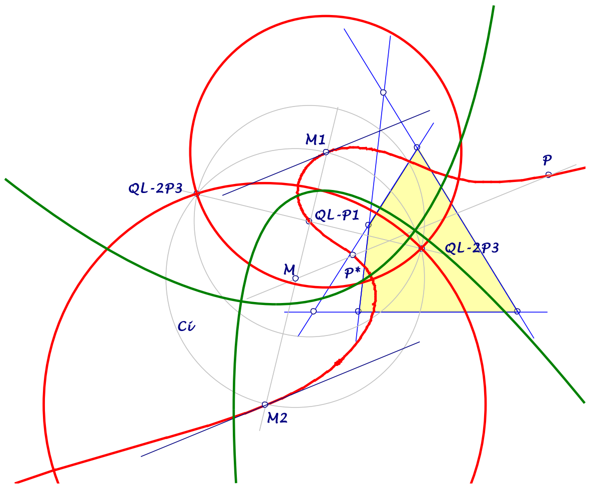


- **A pivotal *PLC*-cubic is an anallagmatic curve.**

The cubic is invariant wrt two inversions. The inversion circles can be constructed as follows:

… *M* : intersection of the 2nd Steiner Axis and *PP**,
… *Ci* : circle round *M* through *QL-2P3*,
… M_i : intersections of *Ci* and the 2nd Steiner Axis.
… Circles round M_i through *QL-2P3* are inversion circles for the cubic.

The tangents in M_i are parallel to the asymptote, that means parallel to *P.P**.

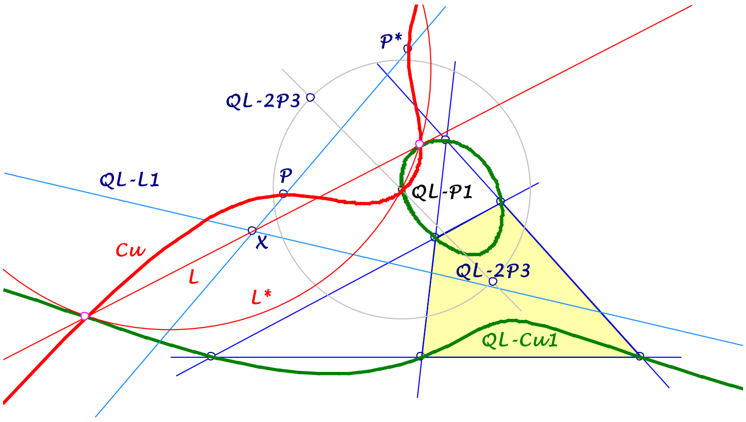


As anallagmatic curve the cubic is twice the envelope of circles, centered on a parabola and orthogonal to one inversion circle. The parabolas have the same focus in *QL-P1* and as directrices the tangents in M_1 and M_2.

- **A pivotal *PLC*-cubic cuts *QL-Cu1* in *QL-P1* and two *CSC*-partners.**

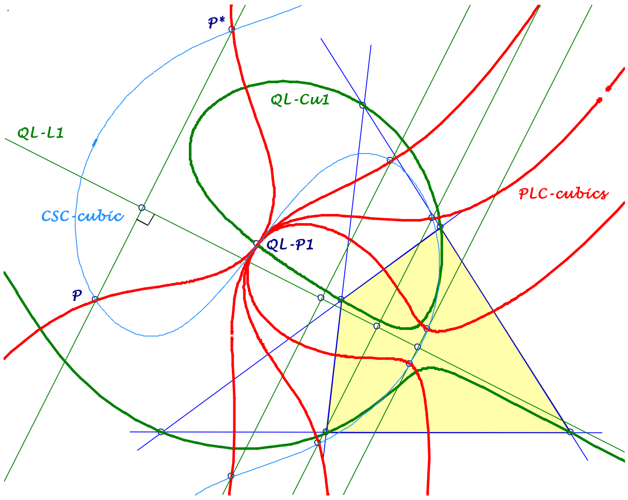
Intersections of pivotal *PLC*-cubics *Cu* and *QL-Cu1* (beside *QL-P1*) can be constructed as follows:

… *X* : intersection of *PP** and *QL-L1*,
… *L* : angle bisector at *X* wrt *QL-2P3*,
… intersections $L \cap L^*$ are the intersections of *Cu* and *QL-Cu1*.



- **All *PLC*-cubics with pivots *P*, *P** and *PP** orthogonal *QL-L1* cut the cubic *QL-Cu1* orthogonal.**

The pivots of these *PLC*-cubics lie on a *QL-P1*-symmetric *CSC*-cubic (see #1237) with pivot in the point at infinity of perpendiculars of *QL-L1*.



Example: Let *P* and *P** be the intersections of *QL-L2* and its *CSC*-circle *QL-Ci3*. The *PLC*-cubic with pivots *P*, *P** intersects the cubic *QL-Cu1* orthogonal in the intersections of the angle bisector at *QL-P7* wrt *QL-2P3* and its *CSC*-circle.

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