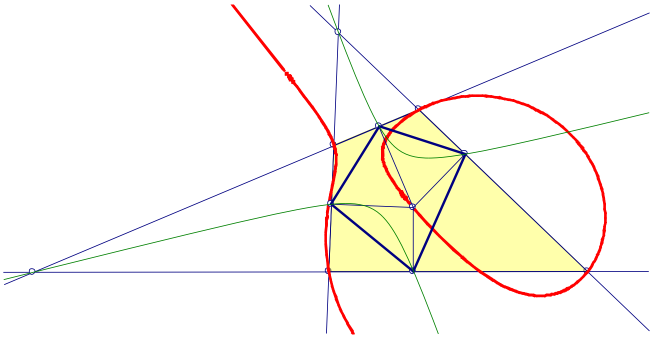
EQF-Note 2015-07-30

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Pedal-Cevian Quadrigons

For a triangle the locus of points, whose pedal triangles are cevian triangles, is the Darboux cubic. Defining cevian quadrigons in a corresponding way there is a pivotal isogonal circular cubic for quadrigons. – Here is a new summary of paper 12.2 on my homepage (see also QFG-message 155).



The vertices of a cevian triangle divide the sides of the reference triangle in ratios with product 1. Analog we can define cevian quadrigons:

Definition: A cevian quadrigon is an inscribed quadrigon, dividing the sides in ratios with product 1.

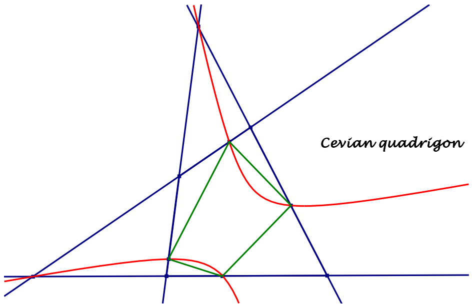
Properties of cevian quadrigons can be found in *09.6* on my homepage. Here some examples:

- **Inscribed conics of a quadrigon give with their contact points cevian quadrigons.**

- **The pedal points for the crosspoints of opposite sides give a cyclic-cevian quadrigon.**

- **Lines from a chosen point to the crosspoints of opposite sides cut the sidelines in vertices of a cevian quadrigon.**

- **The vertices of a cevian quadrigon and the crosspoints of opposite sides lie on a conic …**
  … and conics through the crosspoints generate cevian quadrigons.

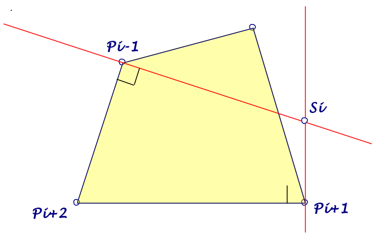


- **The pedal quadrangle of the Miquel Point *S* degenerates collinear to a cevian quadrangle.**

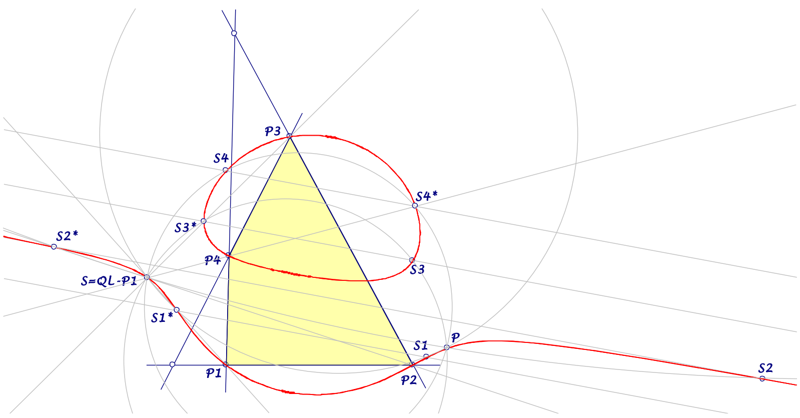
We consider now points, whose pedal quadrigon is a cevian quadrigon. With the 2nd property we can get the equation for a cubic (see *12.2* on my homepage).

> **Theorem: The locus of points, whose pedal quadrigon is a cevian quadrigon, is a cubic through the Miquel point with an asymptote parallel to the bisector of the crosspoints of opposite sides.**

To study the cubic for a quadrangle $P_1P_2P_3P_4$, we use points S_i, for which the pedal points wrt $P_{i-1}P_{i+2}$ and $P_{i+1}P_{i+2}$ are P_{i-1} and P_{i+1}. The conic through the pedal points of S_i and the crosspoints of opposite sides degenerates in two lines.



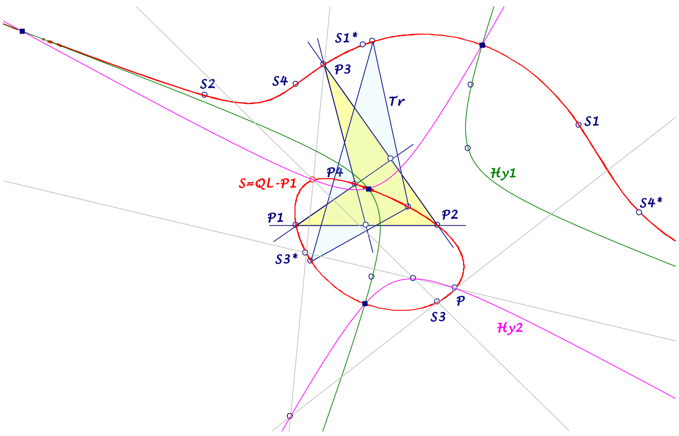
Let L be the bisector of the crosspoints of opposite sides …
… and S_i* the intersection of $S.P_i$ and a parallel to L through S_i
… and P the common point of the circumcircles of S, S_i, S_i*.



- **The points S, S_i, S_i* and P lie on the cubic.**

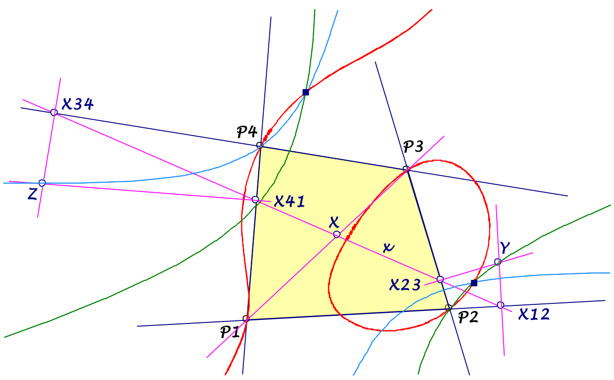
Let Hy_1 be the orthogonal hyperbola through the midpoints of $S_i.S_i*$ …
… and Hy_2 the orthogonal hyperbola through S, P, $S.S_i \cap P.S_i*$, $S.S_i* \cap P.S_i$ …
… and *QA* the orthocentric system of their intersections, …
… which are the in- and excenters of a triangle *Tr*.

- **The triangle *Tr* is the reference triangle for the cubic as pivotal isogonal circular cubic.**



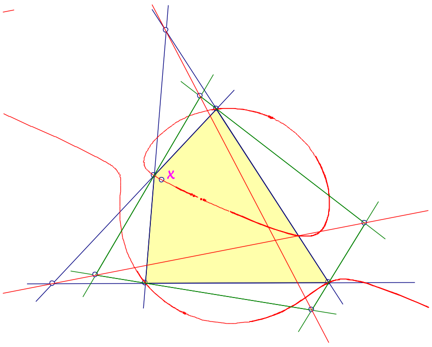
The pivot is the point at infinity for the bisector *L* of the crosspoints of opposite sides of the quadrigon. The used nomination * is isogonal conjugate wrt *Tr*. In this way the cubic can be constructed as locus for intersections of parallels to *L* and their *Tr*-isogonal conjugated image-conic.

On the other hand there is a simple construction without reference triangle and isoconjugation:



Let $P_1P_2P_3P_4$ be the quadrigon. Consider points X on the diagonal $P_1.P_3$ and lines x through X, which cut the sidelines in X_{12}, X_{23}, X_{34}, X_{41}. Let Y be the intersection of perpendiculars in X_{12} and X_{23} to the corresponding sidelines, let Z be the intersection of perpendiculars in X_{34} and X_{41} to the corresponding sidelines. The loci of Y and Z changing the lines x through X are two conics, the intersections of these conics are points of the cubic.

Final property:



- **The cubic is the locus for points *X*, whose antipedal quadrigon has diagonals through the crosspoints of opposite sides of the reference quadrigon.**

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