EQF-Note 2015-10-03

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

(QL-Cu1) for a Quadrangle

Taking for a quadrangle the lines of the Miquel triangle QA-Tr2 and a special further line L, this quadrilateral has a QL-Cu1 with relevant properties in quadrangle geometry.
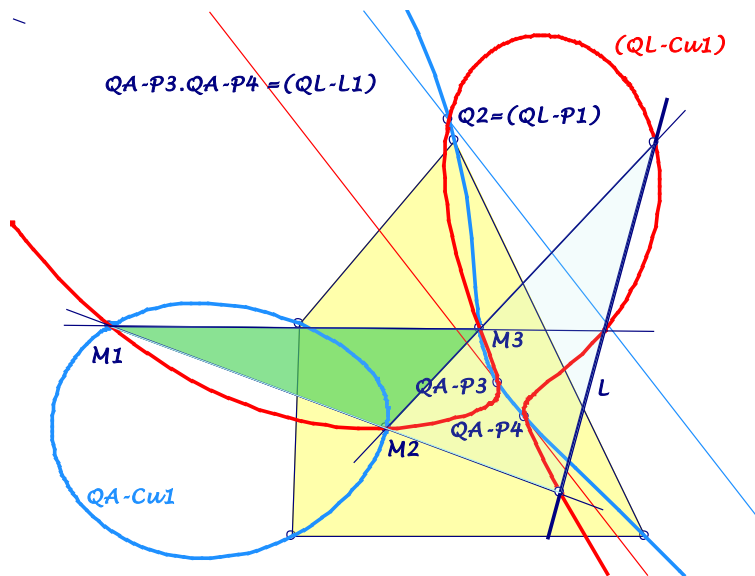
There are similar constructions wrt the following cubics …
  … *QL-Cu*, not in *EQF*, but in *QFG*-message 1308,
  ... *QL-Cu1* for a quadrilateral,
  … *QA-Cu1* for a quadrangle
  … and a new cubic *(QL-Cu1)* for a quadrangle.

QL-Cu is the locus of intersections of …
  … parallels wrt *QL-L1*
  … and their *QL-Tf1*-circles.
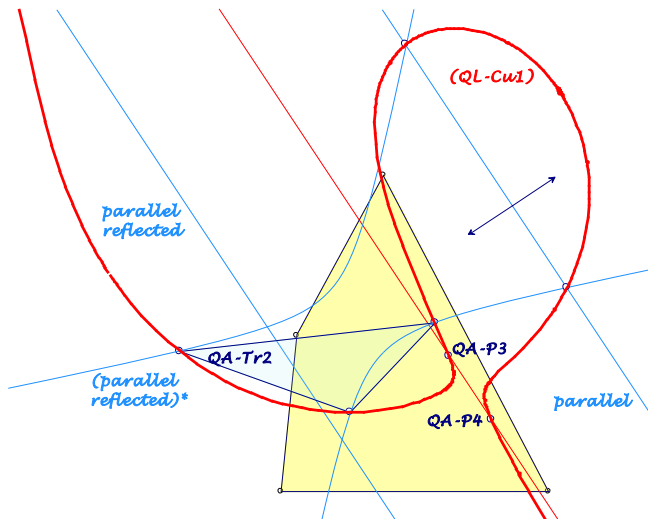
QL-Cu1 is the locus of intersections of …
  … parallels wrt *QL-L1*
  … and the *QL-Tf1*-circles of their reflections in *QL-L1*.

QA-Cu1 is the locus of intersections of …
  … parallels wrt *QA-P3.QA-P4*
  … and their *QA-Tr2*-isogonal conjugate conics.

(QL-Cu1) is the locus of intersections of …
  … parallels wrt *QA-P3.QA-P4*
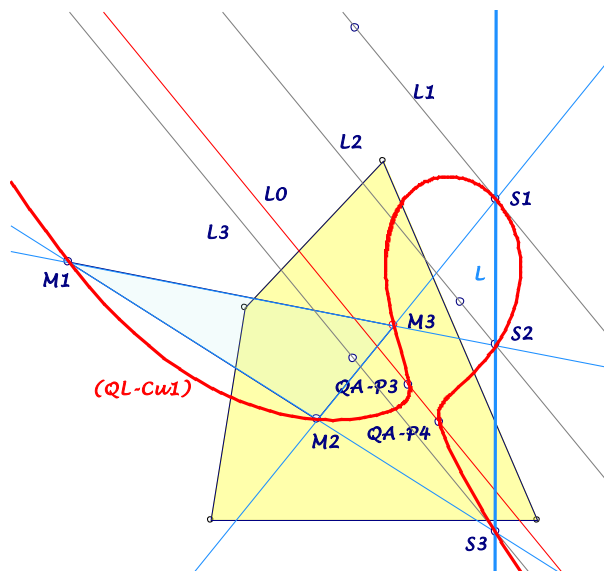 **… and the *QA-Tr2*-isogonal conjugate conics of their reflections in *QA-P3.QA-P4*.**

This cubic *(QL-Cu1)* for a quadrangle is the cubic *QL-Cu1* for a quadrilateral *QL*, which has as three lines the sides of the Miquel triangle *QA-Tr2* and a fourth line *L*, which is the fourth common tangent for all inscribed conics of *QA-Tr2*, which are centered on *QA-P3.QA-P4*. The line *L* can be constructed as follows:

… Let $M_1M_2M_3$ be the Miquel Triangle *QA-Tr2* and $L_0 = $ *QA-P3.QA-P4*.

… Let L_i be a parallel wrt L_0 through the reflection of M_i in L_0.

… Let S_i be the intersection of L_i and M_jM_k.

… $L = S_1S_2S_3$.

- **The cubic *(QL-Cu1)* for a quadrangle is the cubic *QL-Cu1* for the quadrilateral *QL* of the three lines of the Miquel triangle *QA-Tr2* and the line *L*.**

- **The cubic *(QL-Cu1)* for a quadrangle is isogonal invariant wrt the Miquel triangle *QA-Tr2*.**

- **For points on *(QL-Cu1)* the isogonal conjugate wrt the Miquel triangle *QA-Tr2* is the transformation *QL-Tf1* for *QL*.**

- **The cubic *(QL-Cu1)* for a quadrangle is the locus of *QA-Tr2*-isogonal partners with midpoint on *QA-P3.QA-P4*.**

- **The cubic *(QL-Cu1)* for a quadrangle is the locus of the foci of all *QA-Tr2*-inscribed conics centered on *QA-P3.QA-P4*.**

- **The cubic *(QL-Cu1)* for a quadrangle is the locus of points, which have *QA-Tr2*-pedal triangles with circumcenter on *QA-P3.QA-P4*.**

- **The cubic *(QL-Cu1)* for a quadrangle is unipartite with *QA-P3.QA-P4* as Newton line of *QL*.**

- **The cubic *(QL-Cu1)* for a quadrangle contains the following points:**
  … the vertices of the Miquel triangle *QA-Tr2*,
  … the intersection *Q2* of *QA-Cu1* and its asymptote (see *EQF*) as Miquel point *QL-P1* of *QL*,
  … the points *QA-P3, QA-P4* as *QL-2P2* of *QL*,
  … the intersection of the *QA-Cu1*-asymptote and the perpendicular bisector of *QA-P3.QA-P4* as point *T* of *QL* (see *EQF*).

- **The cubics *QA-Cu1* and *(QL-Cu1)* have asymptotes parallel and symmetric wrt *QA-P3.QA-P4*.**

- **The tangent wrt *QA-Cu1* at the intersection *Q2* with its asymptote is *QL-L5* of *QL*.**

- **The *QA-Tr2*-circumcircle is tangent to *(QL-Cu1)* in *Q2*.**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de