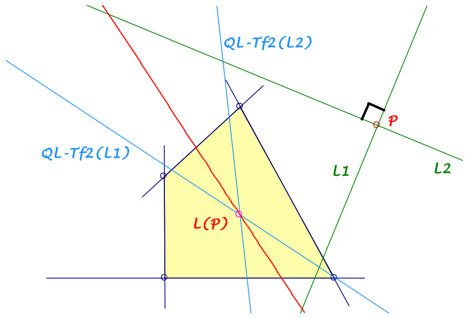
EQF-Note 2015-10-10

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Line Orientated QL-Point Transformation

QL-Tf2 is an isoconjugation for lines, images of orthogonal lines through a point P intersect on a line L(P). The corresponding point-to-line transformation leads in a second step to a point-to-point transformation (e.g. QL-P1 → QL-P17), which maps lines to lines (e.g. QL-L1 → QL-L9).



1. Point-to-Line Transformation *P → L(P)*

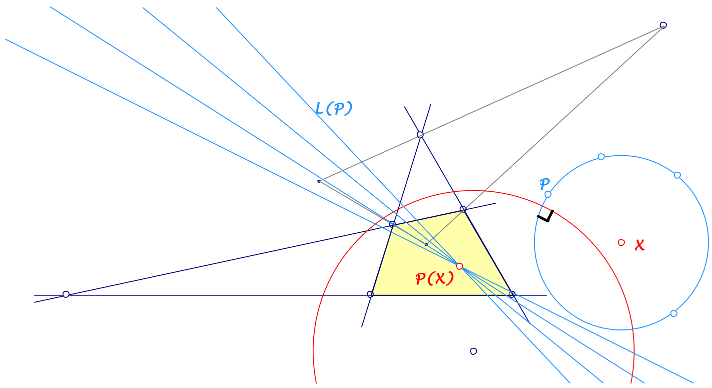
Definition: The image-line *L(P)* for a point *P* is the locus of intersections *QL-Tf2*(L_1) ∩ *QL-Tf2*(L_2) for orthogonal lines L_1 and L_2 through *P*.

This transformation has its own geometry, which shall not be worked out here, but some remarkable properties should be mentioned:

- ***L(QL-P1) = QL-Tf2(QL-L2), L(QL-P10) = QL-L1.***
- ***L(P)* for points *P* on *QL-Cu1* contain the point *P*.**
- **Inverse partners *P* and *Q* wrt the polar circle of *QL-Tr1* have the same line *L(P) = L(Q)*.**
- **For points *P* on a circle, orthogonal to the polar circle of *QL-Tr1*, the lines *L(P)* have a common point.**
- **The locus of all points *P* with line *L(P)* through a fixed point is a circle.**

2. Point-to-Point Transformation $X \rightarrow P(X)$

The last two properties lead to a point-to-point transformation. Let *X* be the center of the circle for the points *P* and let *Y* be the common point of the lines *L(P)*, then we have the transformation $X \rightarrow Y = P(X)$.
If the quadrilateral has an obtuse angled diagonal triangle *QL-Tr1*, the circles for the points *P* are orthogonal to the polar circle of *QL-Tr1*. Only points outside the polar circle have an image.



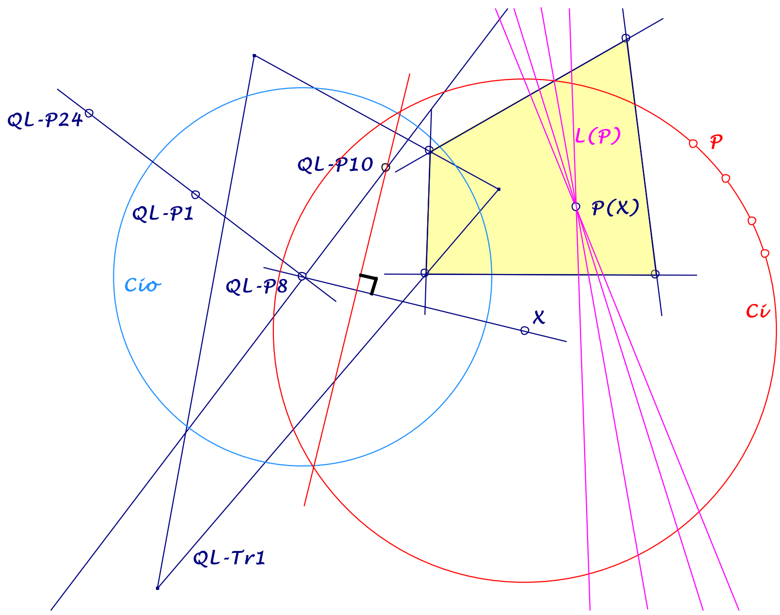
For a construction in general we need a circle …

… Ci_o inversion circle of *QL-P1* and *QL-P24* wrt *QL-P8*.

The image *P(X)* then can be constructed as follows:

Construction of *P(X)*:

> **Let *Ci* be a circle round *X*, which has with Ci_o *a* radical axis through *QL-P10* perpendicular *X.QL-P8*, then the common point of *L(P)* for points *P* on *Ci* is *P(X)*.**



Properties:

- **The transformation *X* → *P(X)* is not self-conjugate, but for iterated applying holds: X_i, X_{i+2}, X_{i+3} are collinear.**

- ***QL-P1* → *QL-P17*, *QL-P8* → *QL-P13*,**
  ***QL-P7* → *QL-L6* ∩ *QL-L9*,**
  ***QL-P19* → point at infinity of *QL-L2*, *QL-L3*.**

- **Points at infinity will be mapped to the Newton line:**
  … of *QL-L1*, *QL-L4* → *QL-P23*,
  … of *QL-L2*, *QL-L3* → *QL-L1* ∩ *QL-L6*,
  … of *QL-L5*, *QL-L6* → midpoint of *QL-P17* and *QL-L6* ∩ *QL-L9*,
  … of *QL-L9* → infinity point of *QL-L1*,
  … of *QL-P1.P8.P24* → *QL-L1* ∩ *QL-P13.P17.P24*.

- **Points on a parallel to *QL-L9* through *QL-P19* will be mapped at infinity.**

If *X* is the intersection of this line and a line *L*, then *P(X)* is the point at infinity of *P(L)*.

- **The transformation *X* → *P(X)* has three fixed points on *QL-Ci6*: S_1, S_2, S_3 (see *EQF*, *QL-Ci6*).**

The triangle $S_1S_2S_3$ is discussed detailed in *QFG*-message 678 and following messages. The vertices are the triple intersections of *QA-Co1* for the three *QG*-components of the quadrilateral. The sidelines are tangent to the *QL*-inscribed parabola *QL-Co1*. The triangle $S_1S_2S_3$ and *QL-Tr1* have a common circumconic through *QL-P8, QL-P13, QL-P24*. Trilinear poles and polars as well as Simson lines and isoconjugations wrt $S_1S_2S_3$ are relevant in *QL*-geometry.

- **For isoconjugations * wrt $S_1S_2S_3$ holds:**
  … The transformation *X* → *P(X*)* = *P(X) is self-conjugate.**
  … The transformation *X* → *P(X*) is the inverse of *X* → *P(X)*.**

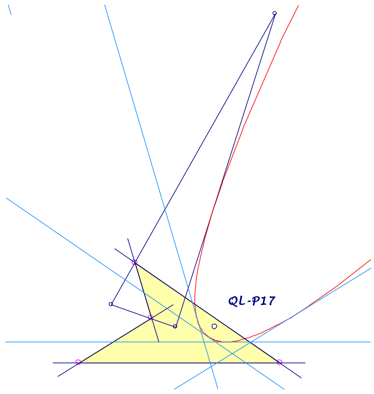
- **Lines will be mapped to lines by *X* → *P(X)*:**
  ***QL-L1* → *QL-L9*, *QL-L2* → *QL-L6*,**
  ***QL-L3* → parallel through *QL-L1* ∩ *QL-L6* to *QL-L3*,**
  ***QL-L4* → perpendicular to *QL-P2.P19* through *QL-P23*,**
  ***QL-L5* → parallel to *QL-L2* through *QL-P17*,**
  ***QL-P1.P8.P24* → *QL-P13.P17.P24*,**

_QL-P1.P25_ → tangent in _QL-P17_ at _QL-Ci1_.

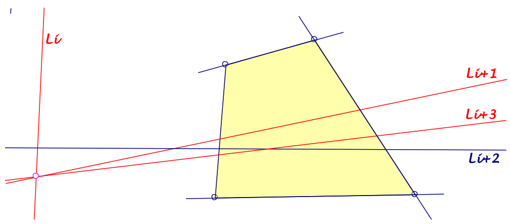
- **Midpoints of the _QL_-diagonals will be mapped in the opposite vertex of _QL-Tr1_.**

- **Diagonals will be mapped to lines through the opposite vertex of _QL-Tr1_ and the intersection of the diagonal with _QL-L1_.**

- **_QL_-lines will be mapped on parallels, tangent to an _QL-Tr1_-inscribed parabola with focus _QL-P17_.**



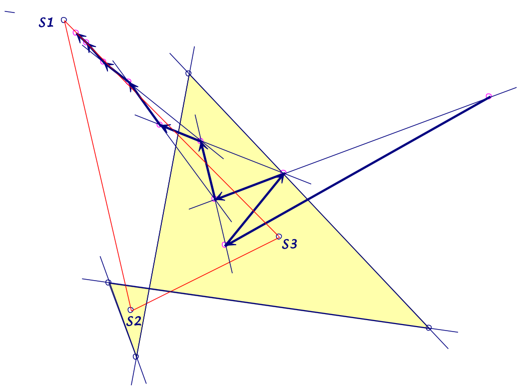
- **For iterated line transformations holds: L_i, L_{i+1}, L_{i+3} have a common point.**



- **For lines _L_ through a point _P_ the intersections of _L_ and _P(L)_ give a circumconic of $S_1S_2S_3$ through _P_:**
  … for _QL-P1_: circle _QL-Ci6_,
  … for _QL-P5_: conic through S_1, S_2, S_3, _QL-P5_ and _QL-P23_,
  … for _QL-P7_: orthogonal hyperbola through S_1, S_2, S_3, _QL-P2_, _QL-P7_, _QL-P23_, _QL-L6_ ∩ _QL-L9_,
  … for _QL-P8_: common circumconic for _QL-Tr1_ and $S_1S_2S_3$ through _QL-P8_, _QL-P13_, _QL-P24_,
  … for _QL-P9_: orthogonal hyperbola through S_1, S_2, S_3, _QL-P2_, _QL-P9_,
  … for _QL-P12_: conic through S_1, S_2, S_3, _QL-P12_, _QL-P23_,
  … for _QL-P20_: conic through S_1, S_2, S_3, _QL-P20_, _QL-P23_,
  … for _QL-P22_: conic through S_1, S_2, S_3, _QL-P22_, _QL-P23_.

- **Circles and conics are mapped to conics:**
  … for *QL-Ci6*: circumconic of $S_1S_2S_3$ through *QL-P1* and *QL-P17*.
  … for *QL-Co1*: *QL-Tr1* inscribed parabola with focus *QL-P17*.

- **Iterated applying of *X* → *P(X)* leads to one of the fixed points S_1, S_2, S_3.**



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