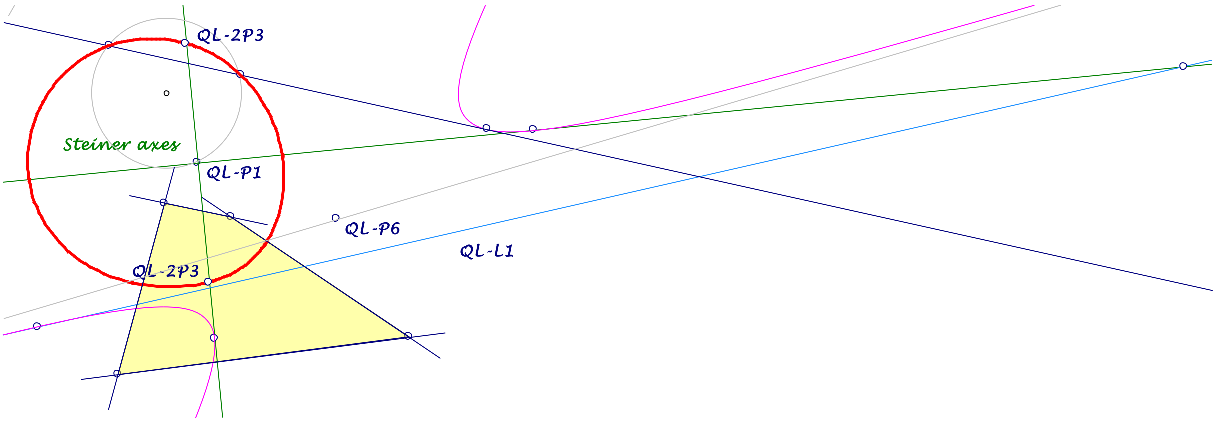
EQF-Note 2015-11-13

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

A Quartic for Quadrilaterals II

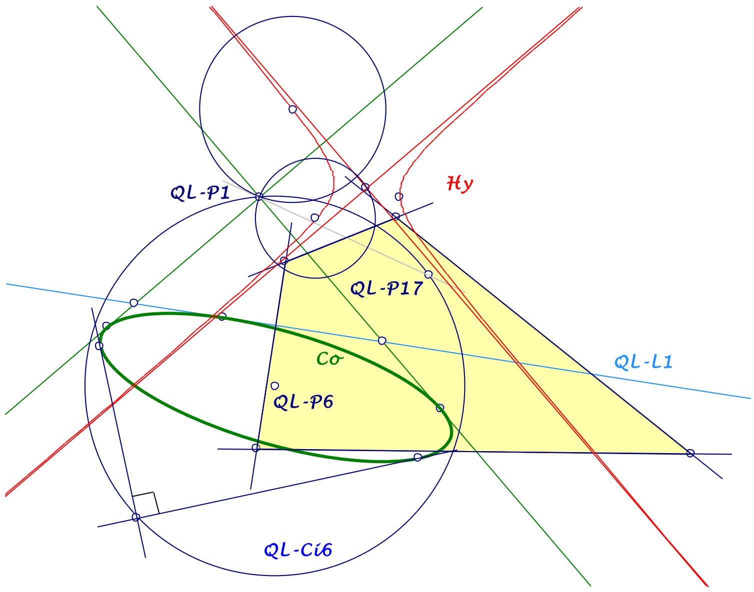
Image partners wrt QL-Tf1 – shortened CSC – on tangents of a conic give a quartic, CSC-invariant through the CSC-fixed points QL-2P3. An example can be found in QFG-message 364. Here another quartic is described with a geometric interpretation orientated at Apollonius circles.



A special conic

Let *Co* be a conic, centered in *QL-P6* and tangent to the Steiner axes and the Newton line *QL-L1*.

- Points on the Dimidium circle *QL-Ci6* have orthogonal tangents wrt the conic *Co*.



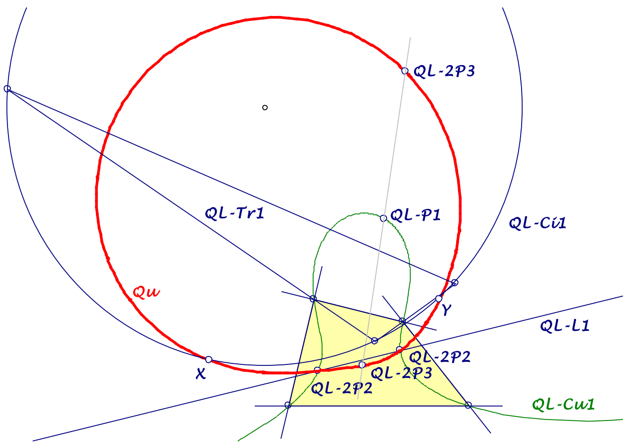
- The *CSC*-circles of tangents at *Co* are centered on an orthogonal hyperbola *Hy* ...
  … through the center of the *CSC*-circle of *QL-L1*,
  … centered in the pole of *QL-P1.QL-P17* wrt *QL-Ci6*,
  … with asymptotes parallel to the Steiner axes,

The Quartic

- *CSC*-partners on tangents of *Co* give a *CSC*-invariant quartic *Qu* through the *CSC*-fixed points *QL-2P3*.

- The quartic *Qu* is also the locus of *CSC*-partners on circles through *QL-P1* centered on the orthogonal hyperbola *Hy*.

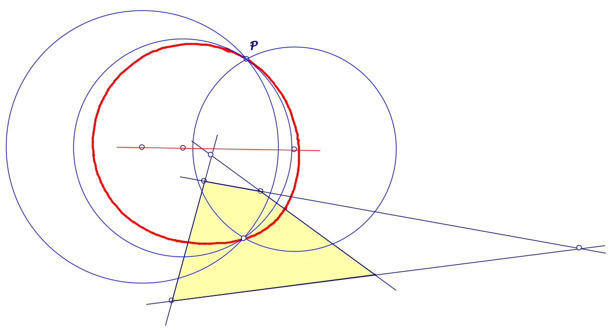
- If the cubic *QL-Cu1* is unipartite, the quartic *Qu* contains the *CSC* conjugated intersections *QL-2P2* of the Newton line and the cubic *QL-Cu1*.



- If the cubic *QL-Cu1* is unipartite, the cubic *Qu* contains the *CSC*-partners *X*, *Y* on the circumcircle *QL-Ci1* of the diagonal triangle.

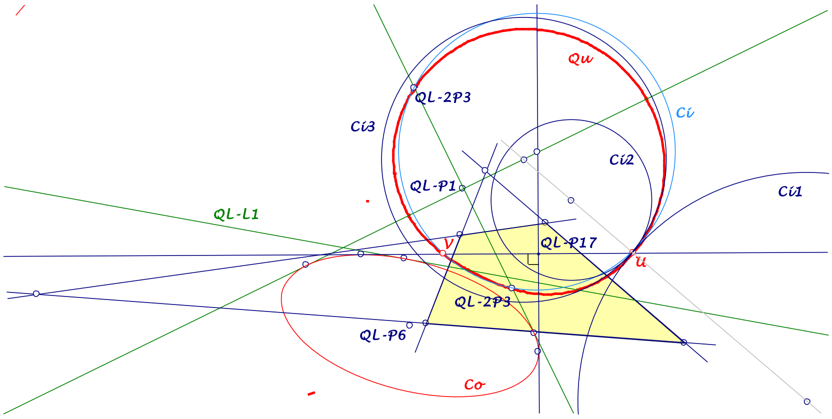
Geometric background

For two opposite points of the quadrilateral we can take the Apollonius circle wrt a given point *P*. There are three such Apollonius circles for a point *P*.



- **The quartic *Qu* is the locus for points *P* with coaxal Apollonius circles.**

Final remark: There are two special points *U*, *V* on the quartic, which are the contact points of their Apollonius circles Ci_1, Ci_2, Ci_3 with a common tangent. These points *U*, *V* can be found as follows: Take the orthogonal tangents from *QL-P17* at the conic *Co*. The *CSC*-partners on one tangent are *U*, *V*. The other tangent has no *CSC*-partners, but cuts the 2nd Steiner axis in the center of a circle *Ci* through *QL-2P3* and *U*, *V*. The common tangent of the Apollonius circles is also tangent to *Co*.



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